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Finite Mathematics

Anders Linnér

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It is all about methodologies!

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The simplest case.

Consider only a finite number of elements.

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Start things by specifying all available elements.

The simplest case.

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Start things by specifying all available elements.

Use list notation.

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The order of the listed elements is not important.

The simplest case.

Consider only a finite number of elements.

Start things by specifying all available elements.

Use list notation.

$$\mathcal{U} = \{a, b, c, d, e, f, g\}.$$

The order of the listed elements is not important.

There should not be any duplicates.

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

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Examples include

$$\{b, c, f, g\}$$

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It is possible to choose all available elements, and the corresponding set $\mathcal{U}$ is known as the universal set.

It is also possible to choose none of the available elements, and the corresponding set is known as the empty set.

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It is possible to choose all available elements, and the corresponding set $\mathcal{U}$ is known as the universal set.

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The empty set is denoted by $\emptyset$, and it has no elements.

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If each element in a set A is also an element of a set B, then write $A \subset B$.

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Observe that $\emptyset \subset A$, $A \subset A$, and $A \subset \mathcal{U}$ no matter what the set A is.

Write $A = B$ whenever $A \subset B$ and $B \subset A$.

The subsets of $\{a, b, c\}$ are

$$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}.$$

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The set of all subsets of a set is known as the power set of the given set.

There are three important ways to create sets from other sets.

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Union, $\cup$, OR.

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Union, $\cup$, **OR.**

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Complement, A^c, **NOT**.

Let $\mathcal{U} = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{b, c, g, i, j\}$, and $B = \{a, b, c, d, e, j\}$.

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Then $A \cup B = \{a, b, c, d, e, g, i, j\}$, $A \cap B = \{b, c, j\}$, $A^c = \{a, d, e, f, h\}$, and $B^c = \{f, g, h, i\}$.

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Then $A \cup B = \{a, b, c, d, e, g, i, j\}$, $A \cap B = \{b, c, j\}$, $A^c = \{a, d, e, f, h\}$, and $B^c = \{f, g, h, i\}$.

Observe that a universal set must be given in order to determine the complement.

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There are a number of set-laws worth knowing.

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$\emptyset^c = \mathcal{U}$
$\mathcal{U}^c = \emptyset$
$(A^c)^c = A$
$B \cup A = A \cup B$
$A \cup (B \cup C) =$
$(A \cup B) \cup C$

$A \cup A = A$
$A \cap A = A$
$A \cup \emptyset = A$
$B \cap A = A \cap B$
$A \cap (B \cap C) =$
$(A \cap B) \cap C$

$A \cap \emptyset = \emptyset$
$A \cup \mathcal{U} = \mathcal{U}$
$A \cap \mathcal{U} = A$

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The next two laws have no counterpart in arithmetic.

The following is true

$$(A \cup B)^c = A^c \cap B^c.$$

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Replace A by A^c and B by B^c, a mere change in naming convention, and rearrange to get

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Replace A by A^c and B by B^c, a mere change in naming convention, and rearrange to get

$$(A \cap B)^c = A^c \cup B^c.$$

These are known as deMorgan's laws.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

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Suppose $x \in (A \cap B)^c$.

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Suppose $x \in (A \cap B)^c$.

It follows that $x \notin A \cap B$.

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Suppose $x \in (A \cap B)^c$.

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It may be that x is in one or the other of the two sets but it is not in both.

If x is not in A then $x \in A^c$ and $x \in A^c \cup B^c$.

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Similarly, if x is not in B then $x \in B^c$ and $x \in A^c \cup B^c$.

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Similarly, if x is not in B then $x \in B^c$ and $x \in A^c \cup B^c$.

In either case $x \in A^c \cup B^c$ and hence $(A \cap B)^c \subset A^c \cup B^c$.

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This time suppose $x \notin (A \cap B)^c$.

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This means x is in $A \cap B$.

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This means $x \notin A^c$ and $x \notin B^c$.

So it follows that $x \notin A^c \cup B^c$ and $(A \cap B)^c \supset A^c \cup B^c$.

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This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

This means x is in $A \cap B$.

It follows that $x \in A$ and $x \in B$.

This means $x \notin A^c$ and $x \notin B^c$.

So it follows that $x \notin A^c \cup B^c$ and $(A \cap B)^c \supset A^c \cup B^c$.

The two set inclusions combine to guarantee that $(A \cap B)^c = A^c \cup B^c$.

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The simplest Venn-diagram consists of a rectangle that represents the universal set of interest, and two ovals that overlap and represent A and B, respectively.

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The universal set splits into four parts: $A \cap B$, $A \cap B^c$, $A^c \cap B$, and $A^c \cap B^c$.

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According to deMorgan, the last set is everything outside $A \cup B$.

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The second kind of Venn-diagram has three ovals corresponding to A, B, and C that are drawn in the the most general position.

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This time the universal set splits into eight parts:

$A \cap B \cap C$ $\quad$ $A \cap B \cap C^c$ $\quad$ $A \cap B^c \cap C^c$ $\quad$ $A^c \cap B^c \cap C^c$

$A \cap B^c \cap C$ $\quad$ $A^c \cap B \cap C^c$

$A^c \cap B \cap C$ $\quad$ $A^c \cap B^c \cap C$

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A subset set created from A, B and C using set-operations involves the inclusion or exclusion of each of the eight parts so there are $2^8 = 256$ possibilities.

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The so-called 'exclusive or', which involves exactly the two parts $A \cap B^c$ and $A^c \cap B$, is represented by $(A \cap B^c) \cup (A^c \cap B)$.

A subset set created from A, B and C using set-operations involves the inclusion or exclusion of each of the eight parts so there are $2^8 = 256$ possibilities.

The Venn-diagram will in each case resolve things as one simply writes a union of the parts involved.

The so-called 'exclusive or', which involves exactly the two parts $A \cap B^c$ and $A^c \cap B$, is represented by $(A \cap B^c) \cup (A^c \cap B)$.

In plain language use, 'one or the other' often means exclusive or.

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Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

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6. By doubling, this creates the sequence, 2^0, 2^1, 2^2, and so on and so forth. □

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In this way infinite sets may be described.

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How many ways are there to fill out a 3 question True–False Exam?
Since there are 2 ways to answer each of the three questions, there are $2^3 = 8$ possible answer keys:

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The multiplication principle asserts that to count the number of ways the task may be completed simply multiply the number of ways each intermediate step may be completed.

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When each object in a finite collection is given a unique designation, then the count of the number of possibilities utilizes the so-called factorial.

Let the collection be $\{a, b, c\}$, and list the members in order.

The selection of the first has three possibilities.

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The selection of the first has three possibilities.

Once the first is selected, only two remain so only two possibilities.

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Write 3! for the product $3 \cdot 2 \cdot 1$, and more generally $1! = 1$ together with $n! = n \cdot (n-1)!$.

Here is a table for the factorial.

3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200
15	1307674368000
16	20922789888000
17	355687428096000

Observe how quickly the values grow.

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Observe that this value is in fact $\frac{8!}{5!}$.

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This suggests defining the permutation function $P(n,r) = \frac{n!}{(n-r)!}$.

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Now it seen that $P(n,n) = P(n,n-1) = n!$, $P(n,1) = n$, and $P(n,0) = 1$.

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The permutation function may also be available on a calculator and it suffers from limitations due to the size of the values produced, just like the factorial.

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'There is only one way to select all objects or none of the objects'.

There are n ways to select exactly one object.

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The formulas are

$$\frac{P(n,1)}{1!} = \frac{n!}{(n-1)! \cdot 1!} = n,$$

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Define the combination function $C(n,r)$ by

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)! \cdot r!}.$$

As seen, by thinking about the objects selected versus the objects left behind it must be that $C(n, r) = C(n, n - r)$.

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But

$$C(2015, 2013) = C(2015, 2015 - 2013) = C(2015, 2),$$

and

$$C(2015, 2) = \frac{P(2015, 2)}{2!} = \frac{2015 \cdot 2014}{2} = 2015 \cdot 1007$$

with

$$2015 \cdot 1007 = 2015(1000 + 7) = 2015000 + 14105 = 2029105.$$

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Now divide out the number of ways of arranging the r selected objects as well as the number of ways of arranging the objects left behind so $\frac{n!}{r!(n-r)!}$, which again is $C(n,r)$.

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This line of thinking works when the objects split into k groups, each containing n_i members so that $n_1+\cdots+n_k=n$.

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This line of thinking works when the objects split into k groups, each containing n_i members so that $n_1+\cdots+n_k=n$.

In this case, with the k designations in order, there are $\frac{n!}{n_1!\cdots n_k!}$ possibilities.

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The question now is how many permutations there are when some of the items are indistinguishable.

To answer that question it suffices to divide by the number of permutations of each collection of indistinguishable objects.

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In the case of S and a, a, a, b, b, c it follows the count is

$$\frac{6!}{3!2!1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 2} = 60.$$

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Now make those placements in each of the following 10 cases: *aaabb*, *aabab*, *aabba*, *abaab*, *ababa*, *abbaa*, *baaab*, *baaba*, *babaa*, and *bbaaa*.

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Recall that $(x+y)^2 = x^2 + 2xy + y^2$ and it follows that

$$(x+y)^3 = (x+y)(x^2+2xy+y^2) = x^3 + 3x^2y + 3xy^2 + y^3.$$

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By thinking about how $(x+y)(x+y)(x+y)$ produces the various terms in the expansion one realizes that the binomial coefficient is involved.

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

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$$1$$

$$1 \quad 1$$

$$1 \; 2 \; 1$$

$$1 \; 3 \; 3 \; 1$$

$$1 \; 4 \; 6 \; 4 \; 1$$

$$1 \; 5 \; 10 \; 10 \; 5 \; 1$$

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

Similarly, $C(3,0) = 1$, $C(3,1) = 3$, $C(3,2)$, and $C(3,3) = 1$.

$$1$$
$$1\ \ 1$$
$$1\ 2\ 1$$
$$1\ 3\ 3\ 1$$
$$1\ 4\ 6\ 4\ 1$$
$$1\,5\,10\,10\,5\,1$$

It follows that $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

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Similarly, $C(3,0) = 1$, $C(3,1) = 3$, $C(3,2)$, and $C(3,3) = 1$.

$$1$$
$$1\ \ 1$$
$$1\ 2\ 1$$
$$1\ 3\ 3\ 1$$
$$1\ 4\ 6\ 4\ 1$$
$$1\,5\,10\,10\,5\,1$$

It follows that $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

Also, $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$.

More generally,

$$(x+y+z)^3 =$$

$$x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3.$$

More generally,

$$(x+y+z)^3 =$$

$$x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3.$$

This time $\frac{3!}{3!0!0!}=1$, $\frac{3!}{2!1!0!}=3$, and $\frac{3!}{1!1!1!}=6$, which are seen as coefficients in the expansion.

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In the binomial case set $x=y=1$ so that $(x+y)^n=(1+1)^n=2^n$.

More generally,

$$(x+y+z)^3 =$$

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This time $\frac{3!}{3!0!0!}=1$, $\frac{3!}{2!1!0!}=3$, and $\frac{3!}{1!1!1!}=6$, which are seen as coefficients in the expansion.

In the binomial case set $x=y=1$ so that $(x+y)^n=(1+1)^n=2^n$.

This implies that

$$C(n,0)+C(n,1)+\cdots+C(n,n-1)+C(n,n)=2^n.$$

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

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This then connects with the fact that the total number of subsets of a set with n elements is 2^n.

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Set the bit to 1 if the element is part of the subset and 0 if it is not.

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This creates a correspondence between subsets of sets with n elements and binary numbers with n digits.

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Set the bit to 1 if the element is part of the subset and 0 if it is not.

This creates a correspondence between subsets of sets with n elements and binary numbers with n digits.

In particular this shows that there are 2^n different binary numbers with n digits.

The first one $00\cdots00$ corresponds to the empty set, whereas the last one $11\cdots11$ corresponds to the whole set.

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Inside computers there is a translation so that $00\cdots00$ corresponds to the number 0, whereas $11\cdots11$ corresponds to the number 2^n-1.

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Inside computers there is a translation so that $00\cdots00$ corresponds to the number 0, whereas $11\cdots11$ corresponds to the number 2^n-1.

In the common case when $n=8$ this means 00000000 corresponds to 0, whereas 11111111 corresponds to $2^8-1=256-1=255$.

The first one $00\cdots00$ corresponds to the empty set, whereas the last one $11\cdots11$ corresponds to the whole set.

Inside computers there is a translation so that $00\cdots00$ corresponds to the number 0, whereas $11\cdots11$ corresponds to the number 2^n-1.

In the common case when $n=8$ this means 00000000 corresponds to 0, whereas 11111111 corresponds to $2^8-1=256-1=255$.

So-called hex-code where $0000\to 0$, $0001\to 1$, ..., $0111\to 7$, $1000\to 8$, $1001\to 9$, $1010\to A$, $1011\to B$, $1100\to C$, $1101\to D$, $1110\to E$, and $1111\to F$, implies that 00 corresponds to 0 and FF corresponds to 255.

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Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

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$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

(c) $A \cap B^c = \{5, 7\}$

Which of the following is true if $A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

(c) $A \cap B^c = \{5, 7\}$

(d) A and B are mutually exclusive.

Which of the following is true if $A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

(c) $A \cap B^c = \{5, 7\}$

(d) A and B are mutually exclusive.

(e) $\{1, 9\} = A \cap B$

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$ so $n(A \cup B) = 7$.

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$ so $n(A \cup B) = 7$.

Observe that $1 \in A$ but not $\{1\} \in A$.

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$ so $n(A \cup B) = 7$.

Observe that $1 \in A$ but not $\{1\} \in A$.

$B^c = \{2, 3, 4, 6, 8\}$ so $A \cap B^c = \{2, 4\}$.

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Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

(c) 12

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Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

(c) 12

(d) 10

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

(c) 12

(d) 10

(e) 7

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

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$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

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Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

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Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

(c) $S = B$, but is not the same as A or C.

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

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Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

(c) $S = B$, but is not the same as A or C.

(d) $S = A = B = C$

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

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$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

(c) $S = B$, but is not the same as A or C.

(d) $S = A = B = C$

(e) $S = C$, but is not the same as A or B.

A book club has a selection of 10 books one month. How many ways are there of picking two books?

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

(c) 2

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

(c) 2

(d) 100

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

(c) 2

(d) 100

(e) 45

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Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) $5 \cdot 4 \cdot 4 \cdot 4$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) $5 \cdot 4 \cdot 4 \cdot 4$

(d) $3 + 2 + 1 + 0$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) $5 \cdot 4 \cdot 4 \cdot 4$

(d) $3 + 2 + 1 + 0$

(e) None of the above.

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How many six-digit telephone numbers are possible if the digits are not all zero?

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

(c) $700,000$

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

(c) $700,000$

(d) 7

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

(c) $700,000$

(d) 7

(e) None of the above.

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What is the value represented by $C(5, 2)$.

What is the value represented by $C(5, 2)$.

(a) 60

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

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What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

(c) 7

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

(c) 7

(d) 5

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

(c) 7

(d) 5

(e) None of the above.

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From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

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From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

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From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

(c) 3375

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From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

(c) 3375

(d) 563

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

(c) 3375

(d) 563

(e) None of these.

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

(c) 450

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A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

(c) 450

(d) 630

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

(c) 450

(d) 630

(e) None of these.

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

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In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

(c) 55

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

(c) 55

(d) 70

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

(c) 55

(d) 70

(e) None of the above.

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Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

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(a) 584

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

(a) 584

(b) 1404

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(c) 820

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(e) 1710

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

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(a) 3

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(a) 3

(b) 97

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(b) 97

(c) $C(100, 12)$

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(c) $C(100, 12)$

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(e) None of the above.

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The compromise is to relinquish the desire to know the behavior of any one instance of the process.

Instead the goal is to understand qualitatively how the process behaves 'in average' when it is repeated many times.

For the purposes here, assume the process results in only a finite number of outcomes.

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This phrase is somewhat misleading since the process need not be thought of as an experiment, nor its evolution as random.

Assume that $n(S) = N$ where N is some integer greater than 1.

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The following is a very useful fact: $Pr(E^c) = 1 - Pr(E)$.

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In a sample space with uniform probability distribution

$$Pr(E) = \frac{n(E)}{n(S)}.$$

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The issue about to be presented is of concern in what is known as statistical mechanics, a subject matter in physics.

Consider the process of tossing two indistinguishable dice simultaneously.

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A reasonable sample space is given by

$$S = \{\{1,1\},\{1,2\},\ldots,\{1,6\},$$

$$\{2,2\},\{2,3\},\ldots,\{2,6\},$$

$$\{3,3\},\{3,4\},\{3,5\},\{3,6\},$$

$$\{4,4\},\{4,5\},\{4,6\},\{5,5\},\{5,6\},\{6,6\}\}.$$

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Observe that $n(S) = 6+5+4+3+2+1 = 21$.

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Observe that $n(S) = 6+5+4+3+2+1 = 21$.

The interpretation of an outcome like $\{2,5\}$ is that one of the dice shows 2 and the other shows 5.

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If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

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When dealing with gas molecules it becomes more of a philosophical question whether such difference is present or not.

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When dealing with gas molecules it becomes more of a philosophical question whether such difference is present or not.

Such a difference, nonetheless, suggests that there are two distinct ways to produce $\{1, 2\}$, whereas there is only a unique way to toss $\{1, 1\}$.

Quantitatively this supports the claim that $p_2 = 2p_1$.

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The event $E =$ 'Toss 11 or higher' has probability $\frac{2}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$.

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As one ponders this kind of issue more deeply there is a connection with the philosophical and religious concept of identity.

Even when it is agreed that two objects, say a pair of hydrogen atoms, cannot be distinguish experimentally, there is still a sense that each has its own identity.

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Assume one is red and the other is green.

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Assume one is red and the other is green.

This time the sample space looks like

$$S' = \{(1,1), \ldots, (1,6), (2,1), \ldots, (2,6), \ldots, (6,1), \ldots, (6,6)\}.$$

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

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Observe that $n(S') = 36$.

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The probability distribution is uniform, and each probability $p_1, \ldots, p_{36}$ is $\frac{1}{36}$.

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This time $E = \{(5, 6), (6, 5), (6, 6)\}$, and $n(E) = 3$, so $Pr(E) = \frac{3}{36} = \frac{1}{12}$.

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This value is equal to the value calculated when the probability distribution is not uniform.

This vindicates the theory since the description of the event in this case only involves the random experiment and not the sample space employed.

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The probabilities must be adjusted upwards to reflect that the new sample space is inside the old sample space.

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

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As expected, $Pr(E|E) = 1$.

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Let E be represented by the region left of the vertical line $x = 1/2$.

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The area of S is 1, and let the area of a region correspond to the probability of the corresponding event.

Let E be represented by the region left of the vertical line $x = 1/2$.

Let F_α be represented by the region below the line from $(0, \alpha)$ to $(1, 1 - \alpha)$ where $0 \leq \alpha \leq 1$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

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Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

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When $\alpha \leq 1/2$, then the rectangle has area $\alpha \times \frac{1}{2}$ and the triangle has area $(\frac{1}{2} - \alpha) \times \frac{1}{2} \times \frac{1}{2}$.

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In either case it is true that $Pr(F_\alpha|E) = \frac{2\alpha+1}{4}$.

When $\alpha = 0$ one gets $\frac{1}{4}$, so the probability of F_α has been reduced from $1/2$ to $1/4$.

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Suppose F is independent of E, then

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{Pr(E)Pr(F|E)}{Pr(F)} = \frac{Pr(E)Pr(F)}{Pr(F)} = Pr(E)$$

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Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

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Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

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It follows that

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It is immediate that the events 'first ball drawn is blue' and 'second ball drawn is blue' are independent.

In fact any pair of the four events are independent.

If this fact is known before one calculates, then $p_1 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, $p_2 = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, $p_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and

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The trouble is that the simplifying impact on the calculations tempts one to assume that the events involved are independent.

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If the example is changed so that there is no replacement after the first draw, then the sample space is $S = \{(r, b), (b, r), (b, b)\}$.

This time $p_1 = p_2 = p_3 = 1/3$, $Pr(E) = Pr(F) = 2/3$, and $Pr(E \cap F) = 1/3$.

Now

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If E and F are independent, it is natural to ask if E^c and F^c are independent, or if E and F^c are independent.

First consider

$$Pr(F^c|E^c) = \frac{Pr(F^c \cap E^c)}{Pr(E^c)} = \frac{Pr((F \cup E)^c)}{Pr(E^c)} =$$

$$\frac{1 - Pr(E \cup F)}{Pr(E^c)} = \frac{1 - Pr(E) - Pr(F) + Pr(E \cap F)}{Pr(E^c)},$$

so

$$Pr(F^c|E^c) = \frac{1 - Pr(E) - Pr(F) + Pr(E)Pr(F)}{1 - Pr(E)} =$$

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It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

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They involve a nice mix of deMorgan's law, and other basic laws of set and probability theory.

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The tree diagram has an initial node from where two edges emanate ending at a node marked D and a node marked D^c.

Both node D and node D^c have a pair of edges emanating to their own pair of nodes marked $+$ and $-$, respectively.

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It is also reasonable to assume that tests are designed so that q is smaller than r.

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The following holds: $Pr(D) = p$, $Pr(D^c) = 1 - p$, $Pr(+|D) = 1 - q$, $Pr(-|D) = q$, $Pr(+|D^c) = r$, and $Pr(-|D^c) = 1 - r$.

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The following 4 quantities are quickly calculated from data in the tree diagram: $Pr(D \cap +) = p(1 - q)$, $Pr(D \cap -) = pq$, $Pr(D^c \cap +) = (1 - p)r$, and $Pr(D^c \cap -) = (1 - p)(1 - r)$.

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With a little bit more work one calculates
$Pr(+) = Pr(D \cap +) + Pr(D^c \cap +) = p(1 - q) + (1 - p)r$, and
$Pr(-) = Pr(D \cap -) + Pr(D^c \cap -) = pq + (1 - p)(1 - r)$.

Finally, there are 4 more additional quantities one may calculate:

$$Pr(D|+) = \frac{Pr(D \cap +)}{Pr(+)} = \frac{p(1-q)}{p(1-q)+(1-p)r},$$

$$Pr(D|-) = \frac{Pr(D \cap -)}{Pr(-)} = \frac{pq}{pq+(1-p)(1-r)},$$

$$Pr(D^c|+) = \frac{Pr(D^c \cap +)}{Pr(+)} = \frac{(1-p)r}{p(1-q)+(1-p)r},$$

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In summary, there are 16 probabilities one may extract from the tree diagram with only some additional basic calculations.

Suppose $p = 2\%$, $q = 1\%$, and $r = 5\%$.

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The exact formula is

$$Pr(D|+) = \frac{2(100-1)}{2(100-1)+(100-2)5} = \frac{198}{198+490} = \frac{198}{688}.$$

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The other approximate formulas are

$$Pr(D|-) \approx 0,$$

$$Pr(D^c|+) \approx \frac{r}{p+r},$$

$$Pr(D^c|-) \approx 1.$$

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The interest in these particular formulas is largely due to the fact that there is very little intuition regarding the values pf $P(D|+)$ and $P(D^c|+)$.

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Rewrite the approximate formulas one more time in the forms

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Observe that $Pr(D|+) + Pr(D^c|+) = 1$ both in the exact and the approximate case.

The approximate formulas supports the view that the crucial quantity in medical testing is the ratio of false positives to the likelihood of having the disease.

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This produces the odd circumstance where the societal concern that sick people spread the disease when given false negatives focuses the attention on q, whereas the individual's desire to assess the likelihood of having the disease focuses the attention on false positives and r.

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The exact formulas produce $\frac{1-q}{2-p-q}$ and $\frac{1-p}{2-p-q}$, so it is seen that this way of 'approximating' is not necessarily completely meaningful.

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The reasoning illustrated in the case $K = L = 2$ generalizes, and the resulting formulas $Pr(E_i|F_j^i)$, sometimes referred to as the a priori probabilities, are given by the so-called Bayes Theorem.

When written in general form these formulas look intimidating, but if in specific case the procedure illustrated by the $K = L = 2$ case is employed, then these formulas follow naturally.