Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Finite Mathematics

Anders Linnér

alinner@math.niu.edu

It is all about methodologies!

Starting Point

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

1 Set Theory

- Finite sets.
- Set notation.
- Universal set and empty set.
- Subsets.
- Sets of sets.
- Set-operations.
- Examples.
- Laws.
- Commutative and associative.
- DeMorgan's laws.
- Proof I.
- Proof II.
- Venn-diagram I.
- Venn-diagram II.
- Exclusive OR.
- Counting.

Starting Point

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

1 Set Theory

- Finite sets.
- Set notation.
- Universal set and empty set.
- Subsets.
- Sets of sets.
- Set-operations.
- Examples.
- Laws.
- Commutative and associative.
- DeMorgan's laws.
- Proof I.
- Proof II.
- Venn-diagram I.
- Venn-diagram II.
- Exclusive OR.
- Counting.

The simplest case.

Consider only a finite number of elements.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Start things by specifying all available elements.

The simplest case.

Consider only a finite number of elements.

Start things by specifying all available elements.

Use list notation.

The simplest case.

Consider only a finite number of elements.

Start things by specifying all available elements.

Use list notation.

$$\mathcal{U} = \{a, b, c, d, e, f, g\}.$$

The simplest case.

Consider only a finite number of elements.

Start things by specifying all available elements.

Use list notation.

$$\mathcal{U} = \{a, b, c, d, e, f, g\}.$$

The order of the listed elements is not important.

The simplest case.

Consider only a finite number of elements.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Start things by specifying all available elements.

Use list notation.

$$\mathcal{U} = \{a, b, c, d, e, f, g\}.$$

The order of the listed elements is not important.

There should not be any duplicates.

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

Examples include

$$\{b, c, f, g\}$$

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

Examples include

$$\{b, c, f, g\}$$

and

$$\{a, e, f\}.$$

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

Examples include

$$\{b, c, f, g\}$$

and

$$\{a, e, f\}.$$

It is possible to choose all available elements, and the corresponding set $\mathcal{U}$ is known as the universal set.

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

Examples include

$$\{b, c, f, g\}$$

and

$$\{a, e, f\}.$$

It is possible to choose all available elements, and the corresponding set $\mathcal{U}$ is known as the universal set.

It is also possible to choose none of the available elements, and the corresponding set is known as the empty set.

Once $\mathcal{U}$ is specified, a set is simply a sample of the available elements.

Examples include

$$\{b, c, f, g\}$$

and

$$\{a, e, f\}.$$

It is possible to choose all available elements, and the corresponding set $\mathcal{U}$ is known as the universal set.

It is also possible to choose none of the available elements, and the corresponding set is known as the empty set.

The empty set is denoted by $\emptyset$, and it has no elements.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

If each element in a set A is also an element of a set B, then write $A \subset B$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

If each element in a set A is also an element of a set B, then write $A \subset B$.

Observe that $\emptyset \subset A$, $A \subset A$, and $A \subset \mathcal{U}$ no matter what the set A is.

If each element in a set A is also an element of a set B, then write $A \subset B$.

Observe that $\emptyset \subset A$, $A \subset A$, and $A \subset \mathcal{U}$ no matter what the set A is.

Write $A = B$ whenever $A \subset B$ and $B \subset A$.

If each element in a set A is also an element of a set B, then write $A \subset B$.

Observe that $\emptyset \subset A$, $A \subset A$, and $A \subset \mathcal{U}$ no matter what the set A is.

Write $A = B$ whenever $A \subset B$ and $B \subset A$.

The subsets of $\{a, b, c\}$ are

$$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}.$$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

The previous listing of subsets looks suspiciously similar to a set.

The previous listing of subsets looks suspiciously similar to a set.

Since it seems reasonable to in fact consider it a set, this vastly broadens the scope of set theory.

The previous listing of subsets looks suspiciously similar to a set.

Since it seems reasonable to in fact consider it a set, this vastly broadens the scope of set theory.

What is the difference between $\emptyset$ and $\{\emptyset\}$?

The previous listing of subsets looks suspiciously similar to a set.

Since it seems reasonable to in fact consider it a set, this vastly broadens the scope of set theory.

What is the difference between $\emptyset$ and $\{\emptyset\}$?

The former has no elements whereas the latter has one element, the empty set.

The previous listing of subsets looks suspiciously similar to a set.

Since it seems reasonable to in fact consider it a set, this vastly broadens the scope of set theory.

What is the difference between $\emptyset$ and $\{\emptyset\}$?

The former has no elements whereas the latter has one element, the empty set.

The set of all subsets of a set is known as the power set of the given set.

There are three important ways to create sets from other sets.

There are three important ways to create sets from other sets.

Union, $\cup$, OR.

There are three important ways to create sets from other sets.

Union, $\cup$, **OR.**

Intersection, $\cap$, **AND.**

There are three important ways to create sets from other sets.

Union, $\cup$, **OR.**

Intersection, $\cap$, **AND.**

Complement, A^c, **NOT**.

Let $\mathcal{U} = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{b, c, g, i, j\}$, and $B = \{a, b, c, d, e, j\}$.

Let $\mathcal{U} = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{b, c, g, i, j\}$, and $B = \{a, b, c, d, e, j\}$.

Then $A \cup B = \{a, b, c, d, e, g, i, j\}$, $A \cap B = \{b, c, j\}$, $A^c = \{a, d, e, f, h\}$, and $B^c = \{f, g, h, i\}$.

Let $\mathcal{U} = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{b, c, g, i, j\}$, and $B = \{a, b, c, d, e, j\}$.

Then $A \cup B = \{a, b, c, d, e, g, i, j\}$, $A \cap B = \{b, c, j\}$, $A^c = \{a, d, e, f, h\}$, and $B^c = \{f, g, h, i\}$.

Observe that a universal set must be given in order to determine the complement.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

There are a number of set-laws worth knowing.

There are a number of set-laws worth knowing.

$\emptyset^c = \mathcal{U}$
$\mathcal{U}^c = \emptyset$
$(A^c)^c = A$
$B \cup A = A \cup B$
$A \cup (B \cup C) =$
$(A \cup B) \cup C$

$A \cup A = A$
$A \cap A = A$
$A \cup \emptyset = A$
$B \cap A = A \cap B$
$A \cap (B \cap C) =$
$(A \cap B) \cap C$

$A \cap \emptyset = \emptyset$
$A \cup \mathcal{U} = \mathcal{U}$
$A \cap \mathcal{U} = A$

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The listed laws do not mix operations.

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The listed laws do not mix operations.

What about the distributive law?

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The listed laws do not mix operations.

What about the distributive law?

There are two distributive laws.

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The listed laws do not mix operations.

What about the distributive law?

There are two distributive laws.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The listed laws do not mix operations.

What about the distributive law?

There are two distributive laws.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

The terminology 'both unions and intersections are commutative as well as associative' is supported by these laws.

The listed laws do not mix operations.

What about the distributive law?

There are two distributive laws.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

The next two laws have no counterpart in arithmetic.

The following is true

$$(A \cup B)^c = A^c \cap B^c.$$

The following is true

$$(A \cup B)^c = A^c \cap B^c.$$

If the claim is correct, then applying the complement to both sides produces $A \cup B = (A^c \cap B^c)^c$.

The following is true

$$(A \cup B)^c = A^c \cap B^c.$$

If the claim is correct, then applying the complement to both sides produces $A \cup B = (A^c \cap B^c)^c$.

Replace A by A^c and B by B^c, a mere change in naming convention, and rearrange to get

$$(A \cap B)^c = A^c \cup B^c.$$

The following is true

$$(A \cup B)^c = A^c \cap B^c.$$

If the claim is correct, then applying the complement to both sides produces $A \cup B = (A^c \cap B^c)^c$.

Replace A by A^c and B by B^c, a mere change in naming convention, and rearrange to get

$$(A \cap B)^c = A^c \cup B^c.$$

These are known as deMorgan's laws.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

Suppose $x \in (A \cap B)^c$.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

Suppose $x \in (A \cap B)^c$.

It follows that $x \notin A \cap B$.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

Suppose $x \in (A \cap B)^c$.

It follows that $x \notin A \cap B$.

It may be that x is in one or the other of the two sets but it is not in both.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

Suppose $x \in (A \cap B)^c$.

It follows that $x \notin A \cap B$.

It may be that x is in one or the other of the two sets but it is not in both.

If x is not in A then $x \in A^c$ and $x \in A^c \cup B^c$.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

Suppose $x \in (A \cap B)^c$.

It follows that $x \notin A \cap B$.

It may be that x is in one or the other of the two sets but it is not in both.

If x is not in A then $x \in A^c$ and $x \in A^c \cup B^c$.

Similarly, if x is not in B then $x \in B^c$ and $x \in A^c \cup B^c$.

It is of course possible to test this claim in some simple examples, but if this is to be true no matter which sets are involved some other reasoning must be employed.

To break things down into smaller steps, first consider the claim that $(A \cap B)^c \subset A^c \cup B^c$.

Suppose $x \in (A \cap B)^c$.

It follows that $x \notin A \cap B$.

It may be that x is in one or the other of the two sets but it is not in both.

If x is not in A then $x \in A^c$ and $x \in A^c \cup B^c$.

Similarly, if x is not in B then $x \in B^c$ and $x \in A^c \cup B^c$.

In either case $x \in A^c \cup B^c$ and hence $(A \cap B)^c \subset A^c \cup B^c$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

This means x is in $A \cap B$.

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

This means x is in $A \cap B$.

It follows that $x \in A$ and $x \in B$.

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

This means x is in $A \cap B$.

It follows that $x \in A$ and $x \in B$.

This means $x \notin A^c$ and $x \notin B^c$.

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

This means x is in $A \cap B$.

It follows that $x \in A$ and $x \in B$.

This means $x \notin A^c$ and $x \notin B^c$.

So it follows that $x \notin A^c \cup B^c$ and $(A \cap B)^c \supset A^c \cup B^c$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Now turn the attention to $(A \cap B)^c \supset A^c \cup B^c$.

This is just different notation for $A^c \cup B^c \subset (A \cap B)^c$.

This time suppose $x \notin (A \cap B)^c$.

This means x is in $A \cap B$.

It follows that $x \in A$ and $x \in B$.

This means $x \notin A^c$ and $x \notin B^c$.

So it follows that $x \notin A^c \cup B^c$ and $(A \cap B)^c \supset A^c \cup B^c$.

The two set inclusions combine to guarantee that $(A \cap B)^c = A^c \cup B^c$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A so-called Venn-diagram may be used to establish less familiar set-identities.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A so-called Venn-diagram may be used to establish less familiar set-identities.

The simplest Venn-diagram consists of a rectangle that represents the universal set of interest, and two ovals that overlap and represent A and B, respectively.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A so-called Venn-diagram may be used to establish less familiar set-identities.

The simplest Venn-diagram consists of a rectangle that represents the universal set of interest, and two ovals that overlap and represent A and B, respectively.

The universal set splits into four parts: $A \cap B$, $A \cap B^c$, $A^c \cap B$, and $A^c \cap B^c$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A so-called Venn-diagram may be used to establish less familiar set-identities.

The simplest Venn-diagram consists of a rectangle that represents the universal set of interest, and two ovals that overlap and represent A and B, respectively.

The universal set splits into four parts: $A \cap B$, $A \cap B^c$, $A^c \cap B$, and $A^c \cap B^c$.

According to deMorgan, the last set is everything outside $A \cup B$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

The second kind of Venn-diagram has three ovals corresponding to A, B, and C that are drawn in the the most general position.

The second kind of Venn-diagram has three ovals corresponding to A, B, and C that are drawn in the the most general position.

This time the universal set splits into eight parts:

$A \cap B \cap C$ $\quad$ $A \cap B \cap C^c$ $\quad$ $A \cap B^c \cap C^c$ $\quad$ $A^c \cap B^c \cap C^c$

$A \cap B^c \cap C$ $\quad$ $A^c \cap B \cap C^c$

$A^c \cap B \cap C$ $\quad$ $A^c \cap B^c \cap C$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A subset set created from A, B and C using set-operations involves the inclusion or exclusion of each of the eight parts so there are $2^8 = 256$ possibilities.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A subset set created from A, B and C using set-operations involves the inclusion or exclusion of each of the eight parts so there are $2^8 = 256$ possibilities.

The Venn-diagram will in each case resolve things as one simply writes a union of the parts involved.

A subset set created from A, B and C using set-operations involves the inclusion or exclusion of each of the eight parts so there are $2^8 = 256$ possibilities.

The Venn-diagram will in each case resolve things as one simply writes a union of the parts involved.

The so-called 'exclusive or', which involves exactly the two parts $A \cap B^c$ and $A^c \cap B$, is represented by $(A \cap B^c) \cup (A^c \cap B)$.

A subset set created from A, B and C using set-operations involves the inclusion or exclusion of each of the eight parts so there are $2^8 = 256$ possibilities.

The Venn-diagram will in each case resolve things as one simply writes a union of the parts involved.

The so-called 'exclusive or', which involves exactly the two parts $A \cap B^c$ and $A^c \cap B$, is represented by $(A \cap B^c) \cup (A^c \cap B)$.

In plain language use, 'one or the other' often means exclusive or.

Write $n(A)$ for the number of elements in the set A.

Write $n(A)$ for the number of elements in the set A.

Observe that $n(\emptyset) = 0$.

Write $n(A)$ for the number of elements in the set A.

Observe that $n(\emptyset) = 0$.

The generally true

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

is connected with the phrase 'inclusion-exclusion'.

Write $n(A)$ for the number of elements in the set A.

Observe that $n(\emptyset) = 0$.

The generally true

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

is connected with the phrase 'inclusion-exclusion'.

A and B are said to be mutually exclusive if $A \cap B = \emptyset$, and in this case $n(A \cup B) = n(A) + n(B)$.

Write $n(A)$ for the number of elements in the set A.

Observe that $n(\emptyset) = 0$.

The generally true

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

is connected with the phrase 'inclusion-exclusion'.

A and B are said to be mutually exclusive if $A \cap B = \emptyset$, and in this case $n(A \cup B) = n(A) + n(B)$.

It also follows that $n(A^c) = n(\mathcal{U}) - n(A)$.

Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

1. Suppose the number of subsets of a set is x.

Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

1. Suppose the number of subsets of a set is x.
2. Now add one extra element to the set.

Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

1. Suppose the number of subsets of a set is x.
2. Now add one extra element to the set.
3. All the previous subsets continue to be subsets.

Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

1. Suppose the number of subsets of a set is x.
2. Now add one extra element to the set.
3. All the previous subsets continue to be subsets.
4. By adding the extra element to each of the previous subsets, another distinct x sets are created for a total of $2x$.

Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

1. Suppose the number of subsets of a set is x.
2. Now add one extra element to the set.
3. All the previous subsets continue to be subsets.
4. By adding the extra element to each of the previous subsets, another distinct x sets are created for a total of $2x$.
5. The empty set has 0 elements and 1 subset, itself.

Theorem

The number of subsets of a finite set A is

$$2^{n(A)}.$$

Proof.

1. Suppose the number of subsets of a set is x.
2. Now add one extra element to the set.
3. All the previous subsets continue to be subsets.
4. By adding the extra element to each of the previous subsets, another distinct x sets are created for a total of $2x$.
5. The empty set has 0 elements and 1 subset, itself.
6. By doubling, this creates the sequence, 2^0, 2^1, 2^2, and so on and so forth. □

The listing of all elements is only practical when the number of elements is fairly small.

The listing of all elements is only practical when the number of elements is fairly small.

There is an alternative notation available.

The listing of all elements is only practical when the number of elements is fairly small.

There is an alternative notation available.

$$\{x \mid 1 \leq x \leq 99,\ x \text{ even integer}\}.$$

The listing of all elements is only practical when the number of elements is fairly small.

There is an alternative notation available.

$$\{x \mid 1 \leq x \leq 99,\ x \text{ even integer}\}.$$

One also writes $x \in \mathbb{Z}$ to indicate that x is an integer.

The listing of all elements is only practical when the number of elements is fairly small.

There is an alternative notation available.

$$\{x \mid 1 \leq x \leq 99,\ x \text{ even integer}\}.$$

One also writes $x \in \mathbb{Z}$ to indicate that x is an integer.

In this way infinite sets may be described.

True False Tests

How many ways are there to fill out a 2 question True–False Exam?

True False Tests

How many ways are there to fill out a 2 question True–False Exam?
Since there are 2 ways to answer the first question and 2 ways to answer the second question, there are $2 \times 2 = 4$ possible answer keys:

$$\text{TT} \quad \text{TF} \quad \text{FT} \quad \text{FF}$$

How many ways are there to fill out a 3 question True–False Exam?

True False Tests

How many ways are there to fill out a 2 question True–False Exam?
Since there are 2 ways to answer the first question and 2 ways to answer the second question, there are $2 \times 2 = 4$ possible answer keys:

$$\text{TT} \quad \text{TF} \quad \text{FT} \quad \text{FF}$$

How many ways are there to fill out a 3 question True–False Exam?
Since there are 2 ways to answer each of the three questions, there are $2^3 = 8$ possible answer keys:

$$\text{TTT} \quad \text{TTF} \quad \text{TFT} \quad \text{TFF} \quad \text{FTT} \quad \text{FTF} \quad \text{FFT} \quad \text{FFF}$$

Given a finite set, a subset A is determined by indicating each of its elements as true, and each other element as true.

Given a finite set, a subset A is determined by indicating each of its elements as true, and each other element as true.

The number of ways this can be done is analogous to the True-False test, so the count is $2^{n(A)}$.

Given a finite set, a subset A is determined by indicating each of its elements as true, and each other element as true.

The number of ways this can be done is analogous to the True-False test, so the count is $2^{n(A)}$.

Consider a task that involve several intermediate steps that must be completed in order.

Given a finite set, a subset A is determined by indicating each of its elements as true, and each other element as true.

The number of ways this can be done is analogous to the True-False test, so the count is $2^{n(A)}$.

Consider a task that involve several intermediate steps that must be completed in order.

To be specific, suppose that all two-character codes of the form 'uppercase letter followed by digit' are to be typed.

Given a finite set, a subset A is determined by indicating each of its elements as true, and each other element as true.

The number of ways this can be done is analogous to the True-False test, so the count is $2^{n(A)}$.

Consider a task that involve several intermediate steps that must be completed in order.

To be specific, suppose that all two-character codes of the form 'uppercase letter followed by digit' are to be typed.

This task has two intermediate steps.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Given a finite set, a subset A is determined by indicating each of its elements as true, and each other element as true.

The number of ways this can be done is analogous to the True-False test, so the count is $2^{n(A)}$.

Consider a task that involve several intermediate steps that must be completed in order.

To be specific, suppose that all two-character codes of the form 'uppercase letter followed by digit' are to be typed.

This task has two intermediate steps.

The multiplication principle asserts that to count the number of ways the task may be completed simply multiply the number of ways each intermediate step may be completed.

In the example the first step may be completed in 26 ways, and the second in 10 ways.

In the example the first step may be completed in 26 ways, and the second in 10 ways.

The number of ways the task may be completed is therefore $26 \cdot 10 = 260$.

In the example the first step may be completed in 26 ways, and the second in 10 ways.

The number of ways the task may be completed is therefore $26 \cdot 10 = 260$.

Observe that even for relatively small values the total count is large enough that listing all the possibilities seems like the wrong approach.

In the example the first step may be completed in 26 ways, and the second in 10 ways.

The number of ways the task may be completed is therefore $26 \cdot 10 = 260$.

Observe that even for relatively small values the total count is large enough that listing all the possibilities seems like the wrong approach.

To aid in these calculations there are several useful mathematical functions.

In the example the first step may be completed in 26 ways, and the second in 10 ways.

The number of ways the task may be completed is therefore $26 \cdot 10 = 260$.

Observe that even for relatively small values the total count is large enough that listing all the possibilities seems like the wrong approach.

To aid in these calculations there are several useful mathematical functions.

When each object in a finite collection is given a unique designation, then the count of the number of possibilities utilizes the so-called factorial.

In the example the first step may be completed in 26 ways, and the second in 10 ways.

The number of ways the task may be completed is therefore $26 \cdot 10 = 260$.

Observe that even for relatively small values the total count is large enough that listing all the possibilities seems like the wrong approach.

To aid in these calculations there are several useful mathematical functions.

When each object in a finite collection is given a unique designation, then the count of the number of possibilities utilizes the so-called factorial.

Let the collection be $\{a, b, c\}$, and list the members in order.

The selection of the first has three possibilities.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

The selection of the first has three possibilities.

Once the first is selected, only two remain so only two possibilities.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

The selection of the first has three possibilities.

Once the first is selected, only two remain so only two possibilities.

For the third there is no choice.

The selection of the first has three possibilities.

Once the first is selected, only two remain so only two possibilities.

For the third there is no choice.

The multiplication principle gives $3 \cdot 2 \cdot 1 = 6$ ways.

The selection of the first has three possibilities.

Once the first is selected, only two remain so only two possibilities.

For the third there is no choice.

The multiplication principle gives $3 \cdot 2 \cdot 1 = 6$ ways.

Write 3! for the product $3 \cdot 2 \cdot 1$, and more generally $1! = 1$ together with $n! = n \cdot (n-1)!$.

Here is a table for the factorial.

3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200
15	1307674368000
16	20922789888000
17	355687428096000

Observe how quickly the values grow.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Observe how quickly the values grow.

Calculators have a factorial key but may not give 14! accurately, let alone 70!.

Observe how quickly the values grow.

Calculators have a factorial key but may not give 14! accurately, let alone 70!.

There are many applications where only some of the available objects are to be given a designation.

Observe how quickly the values grow.

Calculators have a factorial key but may not give 14! accurately, let alone 70!.

There are many applications where only some of the available objects are to be given a designation.

In the 100m dash there are 8 runners competing for first, second, and third.

Observe how quickly the values grow.

Calculators have a factorial key but may not give 14! accurately, let alone 70!.

There are many applications where only some of the available objects are to be given a designation.

In the 100m dash there are 8 runners competing for first, second, and third.

By the multiplication principle, the number of ways the podium may be filled is $8 \cdot 7 \cdot 6 = 336$.

Observe how quickly the values grow.

Calculators have a factorial key but may not give 14! accurately, let alone 70!.

There are many applications where only some of the available objects are to be given a designation.

In the 100m dash there are 8 runners competing for first, second, and third.

By the multiplication principle, the number of ways the podium may be filled is $8 \cdot 7 \cdot 6 = 336$.

Observe that this value is in fact $\frac{8!}{5!}$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

This suggests defining the permutation function $P(n,r) = \frac{n!}{(n-r)!}$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

This suggests defining the permutation function $P(n,r) = \frac{n!}{(n-r)!}$.

In order for this formula to work in complete generality it is necessary to define $0! = 1$, although this at first sight seems far-fetched.

This suggests defining the permutation function $P(n,r) = \frac{n!}{(n-r)!}$.

In order for this formula to work in complete generality it is necessary to define $0! = 1$, although this at first sight seems far-fetched.

Now it seen that $P(n,n) = P(n,n-1) = n!$, $P(n,1) = n$, and $P(n,0) = 1$.

This suggests defining the permutation function $P(n,r) = \frac{n!}{(n-r)!}$.

In order for this formula to work in complete generality it is necessary to define $0! = 1$, although this at first sight seems far-fetched.

Now it seen that $P(n,n) = P(n,n-1) = n!$, $P(n,1) = n$, and $P(n,0) = 1$.

The permutation function may also be available on a calculator and it suffers from limitations due to the size of the values produced, just like the factorial.

There are many times when a selection is made without subsequent designation.

There are many times when a selection is made without subsequent designation.

If each available object is selected, then there is little to say since the selection is viewed as a set and there is one possible set.

There are many times when a selection is made without subsequent designation.

If each available object is selected, then there is little to say since the selection is viewed as a set and there is one possible set.

Technically the calculation is $\frac{P(n,n)}{n!} = \frac{n!}{0!\cdot n!} = 1$.

There are many times when a selection is made without subsequent designation.

If each available object is selected, then there is little to say since the selection is viewed as a set and there is one possible set.

Technically the calculation is $\frac{P(n,n)}{n!} = \frac{n!}{0!\cdot n!} = 1$.

The $n!$ in the denominator accounts for all the ways the n objects may be arranged in order.

There are many times when a selection is made without subsequent designation.

If each available object is selected, then there is little to say since the selection is viewed as a set and there is one possible set.

Technically the calculation is $\frac{P(n,n)}{n!} = \frac{n!}{0!\cdot n!} = 1$.

The $n!$ in the denominator accounts for all the ways the n objects may be arranged in order.

If no object is selected, then again there is not much to say and the formula gives $\frac{P(n,0)}{0!} = \frac{n!}{n!\cdot 0!} = 1$.

There are many times when a selection is made without subsequent designation.

If each available object is selected, then there is little to say since the selection is viewed as a set and there is one possible set.

Technically the calculation is $\frac{P(n,n)}{n!} = \frac{n!}{0!\cdot n!} = 1$.

The $n!$ in the denominator accounts for all the ways the n objects may be arranged in order.

If no object is selected, then again there is not much to say and the formula gives $\frac{P(n,0)}{0!} = \frac{n!}{n!\cdot 0!} = 1$.

'There is only one way to select all objects or none of the objects'.

There are n ways to select exactly one object.

There are n ways to select exactly one object.

There are n ways to select all but one of the objects.

There are n ways to select exactly one object.

There are n ways to select all but one of the objects.

The formulas are

$$\frac{P(n,1)}{1!} = \frac{n!}{(n-1)! \cdot 1!} = n,$$

and

$$\frac{P(n,n-1)}{(n-1)!} = \frac{n!}{1! \cdot (n-1)!} = n.$$

There are n ways to select exactly one object.

There are n ways to select all but one of the objects.

The formulas are

$$\frac{P(n,1)}{1!} = \frac{n!}{(n-1)! \cdot 1!} = n,$$

and

$$\frac{P(n,n-1)}{(n-1)!} = \frac{n!}{1! \cdot (n-1)!} = n.$$

Define the combination function $C(n,r)$ by

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)! \cdot r!}.$$

As seen, by thinking about the objects selected versus the objects left behind it must be that $C(n, r) = C(n, n - r)$.

As seen, by thinking about the objects selected versus the objects left behind it must be that $C(n, r) = C(n, n - r)$.

To illustrate this, $C(2015, 2013)$ would produce hopelessly large values in the formula.

As seen, by thinking about the objects selected versus the objects left behind it must be that $C(n, r) = C(n, n - r)$.

To illustrate this, $C(2015, 2013)$ would produce hopelessly large values in the formula.

But

$$C(2015, 2013) = C(2015, 2015 - 2013) = C(2015, 2),$$

and

$$C(2015, 2) = \frac{P(2015, 2)}{2!} = \frac{2015 \cdot 2014}{2} = 2015 \cdot 1007$$

with

$$2015 \cdot 1007 = 2015(1000 + 7) = 2015000 + 14105 = 2029105.$$

Think of splitting the available objects into two groups, the selected ones and the ones left behind.

Think of splitting the available objects into two groups, the selected ones and the ones left behind.

By the multiplication principle, before adjusting to the indifference about the ordering, the count is $P(n,r) \cdot P(n-r, n-r) = \frac{n!}{(n-r)!} \cdot \frac{(n-r)!}{0!} = n!$.

Think of splitting the available objects into two groups, the selected ones and the ones left behind.

By the multiplication principle, before adjusting to the indifference about the ordering, the count is $P(n,r)\cdot P(n-r,n-r)=\frac{n!}{(n-r)!}\cdot\frac{(n-r)!}{0!}=n!$.

Now divide out the number of ways of arranging the r selected objects as well as the number of ways of arranging the objects left behind so $\frac{n!}{r!(n-r)!}$, which again is $C(n,r)$.

Think of splitting the available objects into two groups, the selected ones and the ones left behind.

By the multiplication principle, before adjusting to the indifference about the ordering, the count is $P(n,r)\cdot P(n-r,n-r)=\frac{n!}{(n-r)!}\cdot\frac{(n-r)!}{0!}=n!$.

Now divide out the number of ways of arranging the r selected objects as well as the number of ways of arranging the objects left behind so $\frac{n!}{r!(n-r)!}$, which again is $C(n,r)$.

This line of thinking works when the objects split into k groups, each containing n_i members so that $n_1+\cdots+n_k=n$.

Think of splitting the available objects into two groups, the selected ones and the ones left behind.

By the multiplication principle, before adjusting to the indifference about the ordering, the count is $P(n,r)\cdot P(n-r,n-r)=\frac{n!}{(n-r)!}\cdot\frac{(n-r)!}{0!}=n!$.

Now divide out the number of ways of arranging the r selected objects as well as the number of ways of arranging the objects left behind so $\frac{n!}{r!(n-r)!}$, which again is $C(n,r)$.

This line of thinking works when the objects split into k groups, each containing n_i members so that $n_1+\cdots+n_k=n$.

In this case, with the k designations in order, there are $\frac{n!}{n_1!\cdots n_k!}$ possibilities.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Now consider the collection a, a, a, b, b, c.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Now consider the collection a, a, a, b, b, c.

This time there are only three distinguishable items: a, b, and c.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Now consider the collection a, a, a, b, b, c.

This time there are only three distinguishable items: a, b, and c.

The number of permutations of the objects in S is $6! = 720$.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Now consider the collection a, a, a, b, b, c.

This time there are only three distinguishable items: a, b, and c.

The number of permutations of the objects in S is $6! = 720$.

One sees the difference between $a_1 a_2 a_3 b_1 b_2 c$ and $a_2 a_1 a_3 b_1 b_2 c$.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Now consider the collection a, a, a, b, b, c.

This time there are only three distinguishable items: a, b, and c.

The number of permutations of the objects in S is $6! = 720$.

One sees the difference between $a_1 a_2 a_3 b_1 b_2 c$ and $a_2 a_1 a_3 b_1 b_2 c$.

If the subscripts are removed then the difference disappear as both look like $aaabbc$.

Consider the objects in $S = \{a_1, a_2, a_3, b_1, b_2, c\}$.

There are 6 distinguishable objects.

Now consider the collection a, a, a, b, b, c.

This time there are only three distinguishable items: a, b, and c.

The number of permutations of the objects in S is $6! = 720$.

One sees the difference between $a_1 a_2 a_3 b_1 b_2 c$ and $a_2 a_1 a_3 b_1 b_2 c$.

If the subscripts are removed then the difference disappear as both look like $aaabbc$.

The question now is how many permutations there are when some of the items are indistinguishable.

To answer that question it suffices to divide by the number of permutations of each collection of indistinguishable objects.

To answer that question it suffices to divide by the number of permutations of each collection of indistinguishable objects.

In the case of S and a, a, a, b, b, c it follows the count is

$$\frac{6!}{3!2!1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 2} = 60.$$

To answer that question it suffices to divide by the number of permutations of each collection of indistinguishable objects.

In the case of S and a, a, a, b, b, c it follows the count is

$$\frac{6!}{3!2!1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 2} = 60.$$

It is important to remember that the order is important so $aaabbc$ is different than $aababc$.

To answer that question it suffices to divide by the number of permutations of each collection of indistinguishable objects.

In the case of S and a, a, a, b, b, c it follows the count is

$$\frac{6!}{3!2!1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 2} = 60.$$

It is important to remember that the order is important so $aaabbc$ is different than $aababc$.

To analyze the formula observe that the c can be placed in anyone of the six positions.

To answer that question it suffices to divide by the number of permutations of each collection of indistinguishable objects.

In the case of S and a, a, a, b, b, c it follows the count is

$$\frac{6!}{3!2!1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 2} = 60.$$

It is important to remember that the order is important so *aaabbc* is different than *aababc*.

To analyze the formula observe that the c can be placed in anyone of the six positions.

Now make those placements in each of the following 10 cases: *aaabb*, *aabab*, *aabba*, *abaab*, *ababa*, *abbaa*, *baaab*, *baaba*, *babaa*, and *bbaaa*.

This produces the 60 possibilities.

This produces the 60 possibilities.

The expression $\frac{n!}{n_1!\cdots n_k!}$ is known as a multinomial coefficient.

This produces the 60 possibilities.

The expression $\frac{n!}{n_1!\cdots n_k!}$ is known as a multinomial coefficient.

In the special case when $k = 2$ it is known as the binomial coefficient.

This produces the 60 possibilities.

The expression $\frac{n!}{n_1!\cdots n_k!}$ is known as a multinomial coefficient.

In the special case when $k = 2$ it is known as the binomial coefficient.

Recall that $(x+y)^2 = x^2 + 2xy + y^2$ and it follows that

$$(x+y)^3 = (x+y)(x^2+2xy+y^2) = x^3 + 3x^2y + 3xy^2 + y^3.$$

This produces the 60 possibilities.

The expression $\frac{n!}{n_1!\cdots n_k!}$ is known as a multinomial coefficient.

In the special case when $k = 2$ it is known as the binomial coefficient.

Recall that $(x+y)^2 = x^2 + 2xy + y^2$ and it follows that

$$(x+y)^3 = (x+y)(x^2+2xy+y^2) = x^3 + 3x^2y + 3xy^2 + y^3.$$

By thinking about how $(x+y)(x+y)(x+y)$ produces the various terms in the expansion one realizes that the binomial coefficient is involved.

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

Similarly, $C(3,0) = 1$, $C(3,1) = 3$, $C(3,2)$, and $C(3,3) = 1$.

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

Similarly, $C(3,0) = 1$, $C(3,1) = 3$, $C(3,2)$, and $C(3,3) = 1$.

$$1$$

$$1 \quad 1$$

$$1 \; 2 \; 1$$

$$1 \; 3 \; 3 \; 1$$

$$1 \; 4 \; 6 \; 4 \; 1$$

$$1 \; 5 \; 10 \; 10 \; 5 \; 1$$

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

Similarly, $C(3,0) = 1$, $C(3,1) = 3$, $C(3,2)$, and $C(3,3) = 1$.

$$1$$
$$1\ \ 1$$
$$1\ 2\ 1$$
$$1\ 3\ 3\ 1$$
$$1\ 4\ 6\ 4\ 1$$
$$1\,5\,10\,10\,5\,1$$

It follows that $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

Observe that $C(2,0) = 1$, $C(2,1) = 2$, and $C(2,2) = 1$.

Similarly, $C(3,0) = 1$, $C(3,1) = 3$, $C(3,2)$, and $C(3,3) = 1$.

$$1$$
$$1\ \ 1$$
$$1\ 2\ 1$$
$$1\ 3\ 3\ 1$$
$$1\ 4\ 6\ 4\ 1$$
$$1\,5\,10\,10\,5\,1$$

It follows that $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

Also, $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$.

More generally,

$$(x+y+z)^3 =$$

$$x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3.$$

More generally,

$$(x+y+z)^3 =$$

$$x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3.$$

This time $\frac{3!}{3!0!0!}=1$, $\frac{3!}{2!1!0!}=3$, and $\frac{3!}{1!1!1!}=6$, which are seen as coefficients in the expansion.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

More generally,

$$(x+y+z)^3 =$$

$$x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3.$$

This time $\frac{3!}{3!0!0!}=1$, $\frac{3!}{2!1!0!}=3$, and $\frac{3!}{1!1!1!}=6$, which are seen as coefficients in the expansion.

In the binomial case set $x=y=1$ so that $(x+y)^n=(1+1)^n=2^n$.

More generally,

$$(x+y+z)^3 =$$

$$x^3+3x^2y+3x^2z+3xy^2+6xyz+3xz^2+y^3+3y^2z+3yz^2+z^3.$$

This time $\frac{3!}{3!0!0!}=1$, $\frac{3!}{2!1!0!}=3$, and $\frac{3!}{1!1!1!}=6$, which are seen as coefficients in the expansion.

In the binomial case set $x=y=1$ so that $(x+y)^n=(1+1)^n=2^n$.

This implies that

$$C(n,0)+C(n,1)+\cdots+C(n,n-1)+C(n,n)=2^n.$$

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

This then connects with the fact that the total number of subsets of a set with n elements is 2^n.

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

This then connects with the fact that the total number of subsets of a set with n elements is 2^n.

A different point of view is provided by the idea of tagging each element by its own bit.

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

This then connects with the fact that the total number of subsets of a set with n elements is 2^n.

A different point of view is provided by the idea of tagging each element by its own bit.

Set the bit to 1 if the element is part of the subset and 0 if it is not.

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

This then connects with the fact that the total number of subsets of a set with n elements is 2^n.

A different point of view is provided by the idea of tagging each element by its own bit.

Set the bit to 1 if the element is part of the subset and 0 if it is not.

This creates a correspondence between subsets of sets with n elements and binary numbers with n digits.

When counting subsets of a set with n elements, the number of subsets with k elements is given by $C(n, k)$.

This then connects with the fact that the total number of subsets of a set with n elements is 2^n.

A different point of view is provided by the idea of tagging each element by its own bit.

Set the bit to 1 if the element is part of the subset and 0 if it is not.

This creates a correspondence between subsets of sets with n elements and binary numbers with n digits.

In particular this shows that there are 2^n different binary numbers with n digits.

The first one $00\cdots00$ corresponds to the empty set, whereas the last one $11\cdots11$ corresponds to the whole set.

The first one $00\cdots00$ corresponds to the empty set, whereas the last one $11\cdots11$ corresponds to the whole set.

Inside computers there is a translation so that $00\cdots00$ corresponds to the number 0, whereas $11\cdots11$ corresponds to the number 2^n-1.

The first one $00\cdots00$ corresponds to the empty set, whereas the last one $11\cdots11$ corresponds to the whole set.

Inside computers there is a translation so that $00\cdots00$ corresponds to the number 0, whereas $11\cdots11$ corresponds to the number 2^n-1.

In the common case when $n=8$ this means 00000000 corresponds to 0, whereas 11111111 corresponds to $2^8-1=256-1=255$.

The first one $00\cdots00$ corresponds to the empty set, whereas the last one $11\cdots11$ corresponds to the whole set.

Inside computers there is a translation so that $00\cdots00$ corresponds to the number 0, whereas $11\cdots11$ corresponds to the number 2^n-1.

In the common case when $n=8$ this means 00000000 corresponds to 0, whereas 11111111 corresponds to $2^8-1=256-1=255$.

So-called hex-code where $0000\to 0$, $0001\to 1$, ..., $0111\to 7$, $1000\to 8$, $1001\to 9$, $1010\to A$, $1011\to B$, $1100\to C$, $1101\to D$, $1110\to E$, and $1111\to F$, implies that 00 corresponds to 0 and FF corresponds to 255.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

Which of the following is true if
$A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

(c) $A \cap B^c = \{5, 7\}$

Which of the following is true if $A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

(c) $A \cap B^c = \{5, 7\}$

(d) A and B are mutually exclusive.

Which of the following is true if $A = \{1, 2, 4, 6, 9\}$, $B = \{1, 5, 7, 9\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

(a) $\mathrm{n}(A \cup B) = 9$

(b) $\{1\} \in A$

(c) $A \cap B^c = \{5, 7\}$

(d) A and B are mutually exclusive.

(e) $\{1, 9\} = A \cap B$

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$ so $n(A \cup B) = 7$.

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$ so $n(A \cup B) = 7$.

Observe that $1 \in A$ but not $\{1\} \in A$.

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$ so $n(A \cup B) = 7$.

Observe that $1 \in A$ but not $\{1\} \in A$.

$B^c = \{2, 3, 4, 6, 8\}$ so $A \cap B^c = \{2, 4\}$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

(c) 12

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

(c) 12

(d) 10

Let $U = \{1, 2, 3, 4, 5, 6, a, b, c, d, e\}$ and $A = \{1, 2, a, e\}$ then $n(A^c) =$

(a) 4

(b) 6

(c) 12

(d) 10

(e) 7

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

(c) $S = B$, but is not the same as A or C.

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

(c) $S = B$, but is not the same as A or C.

(d) $S = A = B = C$

Let $S = \{a, e, l, t, r\}$ and consider:

$$A = \{x | x \text{ is a letter in } late\},$$

$$B = \{x | x \text{ is a letter in } elate\},$$

$$C = \{x | x \text{ is a letter in } latter\}.$$

Which statement is true?

(a) S, A, B, C are distinct sets.

(b) $S = A$, but is not the same as B or C.

(c) $S = B$, but is not the same as A or C.

(d) $S = A = B = C$

(e) $S = C$, but is not the same as A or B.

A book club has a selection of 10 books one month. How many ways are there of picking two books?

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

(c) 2

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

(c) 2

(d) 100

A book club has a selection of 10 books one month. How many ways are there of picking two books?

(a) 90

(b) 50

(c) 2

(d) 100

(e) 45

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) $5 \cdot 4 \cdot 4 \cdot 4$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) $5 \cdot 4 \cdot 4 \cdot 4$

(d) $3 + 2 + 1 + 0$

Five houses in a row are to be painted either Red, Blue, Green or Yellow. In how many ways can this be done if adjacent houses are to be painted different colors?

(a) $5 \cdot 4 \cdot 3 \cdot 2$

(b) $4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) $5 \cdot 4 \cdot 4 \cdot 4$

(d) $3 + 2 + 1 + 0$

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

How many six-digit telephone numbers are possible if the digits are not all zero?

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

(c) $700,000$

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

(c) $700,000$

(d) 7

How many six-digit telephone numbers are possible if the digits are not all zero?

(a) $999,999$

(b) $900,000$

(c) $700,000$

(d) 7

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

What is the value represented by $C(5, 2)$.

What is the value represented by $C(5, 2)$.

(a) 60

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

(c) 7

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

(c) 7

(d) 5

What is the value represented by $C(5, 2)$.

(a) 60

(b) 20

(c) 7

(d) 5

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

(c) 3375

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

(c) 3375

(d) 563

From a department of 15 people, how many ways are there of choosing a committee consisting of a president, a vice president, and a secretary?

(a) 2730

(b) 455

(c) 3375

(d) 563

(e) None of these.

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

(c) 450

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

(c) 450

(d) 630

A child's toy consists of a head, torso, legs and arms. If there are 5 choices of head, 7 for torso, 3 for legs, and 6 for arms, how many different toys are possible?

(a) 105

(b) 126

(c) 450

(d) 630

(e) None of these.

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

(c) 55

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

(c) 55

(d) 70

In a class of 100 students, 10 jog, 25 swim and 5 both jog and swim. How many neither jog nor swim?

(a) 65

(b) 60

(c) 55

(d) 70

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

(a) 584

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

(a) 584

(b) 1404

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

(a) 584

(b) 1404

(c) 820

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

(a) 584

(b) 1404

(c) 820

(d) 1126

Out of 2530 subscribers of a local newspaper, 1946 take the paper daily and 820 take both the daily and Sunday editions. How many subscribe to the Sunday edition?

(a) 584

(b) 1404

(c) 820

(d) 1126

(e) 1710

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can this be done without choosing a rotten egg?

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can this be done without choosing a rotten egg?

(a) 3

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can this be done without choosing a rotten egg?

(a) 3

(b) 97

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can this be done without choosing a rotten egg?

(a) 3

(b) 97

(c) $C(100, 12)$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can this be done without choosing a rotten egg?

(a) 3

(b) 97

(c) $C(100, 12)$

(d) $C(97, 12)$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can this be done without choosing a rotten egg?

(a) 3

(b) 97

(c) $C(100, 12)$

(d) $C(97, 12)$

(e) None of the above.

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can the customer get at most 1 rotten egg?

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can the customer get at most 1 rotten egg?

(a) $C(97, 12) + 3 \cdot C(97, 11)$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can the customer get at most 1 rotten egg?

(a) $C(97, 12) + 3 \cdot C(97, 11)$

(b) 100

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can the customer get at most 1 rotten egg?

(a) $C(97, 12) + 3 \cdot C(97, 11)$

(b) 100

(c) $3 \cdot C(100, 11)$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can the customer get at most 1 rotten egg?

(a) $C(97, 12) + 3 \cdot C(97, 11)$

(b) 100

(c) $3 \cdot C(100, 11)$

(d) $3 \cdot C(97, 11)$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

In how many ways can the customer get at most 1 rotten egg?

(a) $C(97, 12) + 3 \cdot C(97, 11)$

(b) 100

(c) $3 \cdot C(100, 11)$

(d) $3 \cdot C(97, 11)$

(e) None of the above.

The starting point is the desire to analyze a repetitive process that is so intricate that a full-fledged model would be beyond any hope to quantitatively describe.

The starting point is the desire to analyze a repetitive process that is so intricate that a full-fledged model would be beyond any hope to quantitatively describe.

A good example to keep in mind is the toss of a standard six-sided die.

The starting point is the desire to analyze a repetitive process that is so intricate that a full-fledged model would be beyond any hope to quantitatively describe.

A good example to keep in mind is the toss of a standard six-sided die.

The dynamical action by the contact forces at point of release is by itself so complicated that an adequate model of the toss is not attainable.

The starting point is the desire to analyze a repetitive process that is so intricate that a full-fledged model would be beyond any hope to quantitatively describe.

A good example to keep in mind is the toss of a standard six-sided die.

The dynamical action by the contact forces at point of release is by itself so complicated that an adequate model of the toss is not attainable.

The compromise is to relinquish the desire to know the behavior of any one instance of the process.

The starting point is the desire to analyze a repetitive process that is so intricate that a full-fledged model would be beyond any hope to quantitatively describe.

A good example to keep in mind is the toss of a standard six-sided die.

The dynamical action by the contact forces at point of release is by itself so complicated that an adequate model of the toss is not attainable.

The compromise is to relinquish the desire to know the behavior of any one instance of the process.

Instead the goal is to understand qualitatively how the process behaves 'in average' when it is repeated many times.

For the purposes here, assume the process results in only a finite number of outcomes.

For the purposes here, assume the process results in only a finite number of outcomes.

The outcomes are collected into a set known as the sample space.

For the purposes here, assume the process results in only a finite number of outcomes.

The outcomes are collected into a set known as the sample space.

In the toss-a-die process the sample space may be given as $S = \{1, 2, 3, 4, 5, 6\}$.

For the purposes here, assume the process results in only a finite number of outcomes.

The outcomes are collected into a set known as the sample space.

In the toss-a-die process the sample space may be given as $S = \{1, 2, 3, 4, 5, 6\}$.

It is possible to use different labels for the outcomes, but the more obvious the connection is to the process the better.

For the purposes here, assume the process results in only a finite number of outcomes.

The outcomes are collected into a set known as the sample space.

In the toss-a-die process the sample space may be given as $S = \{1, 2, 3, 4, 5, 6\}$.

It is possible to use different labels for the outcomes, but the more obvious the connection is to the process the better.

When the process is subjected to the techniques about to be developed it is referred to as a random experiment.

For the purposes here, assume the process results in only a finite number of outcomes.

The outcomes are collected into a set known as the sample space.

In the toss-a-die process the sample space may be given as $S = \{1, 2, 3, 4, 5, 6\}$.

It is possible to use different labels for the outcomes, but the more obvious the connection is to the process the better.

When the process is subjected to the techniques about to be developed it is referred to as a random experiment.

This phrase is somewhat misleading since the process need not be thought of as an experiment, nor its evolution as random.

Assume that $n(S) = N$ where N is some integer greater than 1.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

A probability distribution on S is a collection of numbers $\{p_1, \ldots, p_N\}$ where each number is in $[0, 1]$ and $p_1 + \cdots + p_N = 1$.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

A probability distribution on S is a collection of numbers $\{p_1, \ldots, p_N\}$ where each number is in $[0, 1]$ and $p_1 + \cdots + p_N = 1$.

An event E is a subset of S, so $E \subset S$ holds.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

A probability distribution on S is a collection of numbers $\{p_1, \ldots, p_N\}$ where each number is in $[0, 1]$ and $p_1 + \cdots + p_N = 1$.

An event E is a subset of S, so $E \subset S$ holds.

Write $Pr(E)$ for the probability of an event.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

A probability distribution on S is a collection of numbers $\{p_1, \ldots, p_N\}$ where each number is in $[0, 1]$ and $p_1 + \cdots + p_N = 1$.

An event E is a subset of S, so $E \subset S$ holds.

Write $Pr(E)$ for the probability of an event.

The definition is that $Pr(E)$ is the sum of all the p_i with $s_i \in E$.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

A probability distribution on S is a collection of numbers $\{p_1, \ldots, p_N\}$ where each number is in $[0, 1]$ and $p_1 + \cdots + p_N = 1$.

An event E is a subset of S, so $E \subset S$ holds.

Write $Pr(E)$ for the probability of an event.

The definition is that $Pr(E)$ is the sum of all the p_i with $s_i \in E$.

It follows that $0 \leq Pr(E) \leq 1$, $Pr(\emptyset) = 0$, and $Pr(S) = 1$.

Assume that $n(S) = N$ where N is some integer greater than 1.

Denote the outcomes by $S = \{s_1, \ldots, s_N\}$.

A probability distribution on S is a collection of numbers $\{p_1, \ldots, p_N\}$ where each number is in $[0, 1]$ and $p_1 + \cdots + p_N = 1$.

An event E is a subset of S, so $E \subset S$ holds.

Write $Pr(E)$ for the probability of an event.

The definition is that $Pr(E)$ is the sum of all the p_i with $s_i \in E$.

It follows that $0 \leq Pr(E) \leq 1$, $Pr(\emptyset) = 0$, and $Pr(S) = 1$.

The following is a very useful fact: $Pr(E^c) = 1 - Pr(E)$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

If E and F are events, then
$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$.

If E and F are events, then
$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$.

In particular, if E and F are mutually exclusive, then
$Pr(E \cup F) = Pr(E) + Pr(F)$.

If E and F are events, then
$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$.

In particular, if E and F are mutually exclusive, then
$Pr(E \cup F) = Pr(E) + Pr(F)$.

If $p_1 = \cdots = p_N = \frac{1}{N}$, then the probability distribution is said to be uniform.

If E and F are events, then
$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$.

In particular, if E and F are mutually exclusive, then
$Pr(E \cup F) = Pr(E) + Pr(F)$.

If $p_1 = \cdots = p_N = \frac{1}{N}$, then the probability distribution is said to be uniform.

Calculations of probabilities are considerably simplified in spaces with uniform probability distribution.

If E and F are events, then
$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$.

In particular, if E and F are mutually exclusive, then
$Pr(E \cup F) = Pr(E) + Pr(F)$.

If $p_1 = \cdots = p_N = \frac{1}{N}$, then the probability distribution is said to be uniform.

Calculations of probabilities are considerably simplified in spaces with uniform probability distribution.

In a sample space with uniform probability distribution

$$Pr(E) = \frac{n(E)}{n(S)}.$$

A fair die has uniform probability distribution.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

Suppose a die is 'biased', 'unfair', or 'loaded' so that $p_6 = 1/5$.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

Suppose a die is 'biased', 'unfair', or 'loaded' so that $p_6 = 1/5$.

Assume $p_1 = \cdots = p_5 = x$ so that $5x + \frac{1}{5} = 1$.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

Suppose a die is 'biased', 'unfair', or 'loaded' so that $p_6 = 1/5$.

Assume $p_1 = \cdots = p_5 = x$ so that $5x + \frac{1}{5} = 1$.

It follows that $5x = 4/5$ and $x = 4/25$.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

Suppose a die is 'biased', 'unfair', or 'loaded' so that $p_6 = 1/5$.

Assume $p_1 = \cdots = p_5 = x$ so that $5x + \frac{1}{5} = 1$.

It follows that $5x = 4/5$ and $x = 4/25$.

This time $Pr(E) = 4/25 + 1/5 = 9/25$, and $27/75 > 25/75$, which is a subtle but noticeable difference.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

Suppose a die is 'biased', 'unfair', or 'loaded' so that $p_6 = 1/5$.

Assume $p_1 = \cdots = p_5 = x$ so that $5x + \frac{1}{5} = 1$.

It follows that $5x = 4/5$ and $x = 4/25$.

This time $Pr(E) = 4/25 + 1/5 = 9/25$, and $27/75 > 25/75$, which is a subtle but noticeable difference.

In some circumstances it is possible to assign several different sample spaces to the same random experiment.

A fair die has uniform probability distribution.

If $E =$ 'Toss 5 or 6', then $n(E) = 2$ and $n(S) = 6$ so $Pr(E) = 2/6 = 1/3$.

Suppose a die is 'biased', 'unfair', or 'loaded' so that $p_6 = 1/5$.

Assume $p_1 = \cdots = p_5 = x$ so that $5x + \frac{1}{5} = 1$.

It follows that $5x = 4/5$ and $x = 4/25$.

This time $Pr(E) = 4/25 + 1/5 = 9/25$, and $27/75 > 25/75$, which is a subtle but noticeable difference.

In some circumstances it is possible to assign several different sample spaces to the same random experiment.

The issue about to be presented is of concern in what is known as statistical mechanics, a subject matter in physics.

Consider the process of tossing two indistinguishable dice simultaneously.

Consider the process of tossing two indistinguishable dice simultaneously.

A reasonable sample space is given by

$$S = \{\{1,1\},\{1,2\},\ldots,\{1,6\},$$

$$\{2,2\},\{2,3\},\ldots,\{2,6\},$$

$$\{3,3\},\{3,4\},\{3,5\},\{3,6\},$$

$$\{4,4\},\{4,5\},\{4,6\},\{5,5\},\{5,6\},\{6,6\}\}.$$

Consider the process of tossing two indistinguishable dice simultaneously.

A reasonable sample space is given by

$$S = \{\{1,1\},\{1,2\},\ldots,\{1,6\},$$

$$\{2,2\},\{2,3\},\ldots,\{2,6\},$$

$$\{3,3\},\{3,4\},\{3,5\},\{3,6\},$$

$$\{4,4\},\{4,5\},\{4,6\},\{5,5\},\{5,6\},\{6,6\}\}.$$

Observe that $n(S) = 6+5+4+3+2+1 = 21$.

Consider the process of tossing two indistinguishable dice simultaneously.

A reasonable sample space is given by

$$S = \{\{1,1\},\{1,2\},\ldots,\{1,6\},$$

$$\{2,2\},\{2,3\},\ldots,\{2,6\},$$

$$\{3,3\},\{3,4\},\{3,5\},\{3,6\},$$

$$\{4,4\},\{4,5\},\{4,6\},\{5,5\},\{5,6\},\{6,6\}\}.$$

Observe that $n(S) = 6+5+4+3+2+1 = 21$.

The interpretation of an outcome like $\{2,5\}$ is that one of the dice shows 2 and the other shows 5.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

It is also reasonable to assume that the probability value when the two dice show different values should also all be the same.

If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

It is also reasonable to assume that the probability value when the two dice show different values should also all be the same.

The intriguing question is whether $p_1 = p_2$.

If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

It is also reasonable to assume that the probability value when the two dice show different values should also all be the same.

The intriguing question is whether $p_1 = p_2$.

In the case of dice it may be argued that a possibly very close examination of the two dice would reveal, however minuscule, a difference.

If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

It is also reasonable to assume that the probability value when the two dice show different values should also all be the same.

The intriguing question is whether $p_1 = p_2$.

In the case of dice it may be argued that a possibly very close examination of the two dice would reveal, however minuscule, a difference.

When dealing with gas molecules it becomes more of a philosophical question whether such difference is present or not.

If it is assumed that both dice are fair, then $p_1 = p_7 = p_{12} = p_{16} = p_{19} = p_{21}$ since in each case the two dice show the same value.

It is also reasonable to assume that the probability value when the two dice show different values should also all be the same.

The intriguing question is whether $p_1 = p_2$.

In the case of dice it may be argued that a possibly very close examination of the two dice would reveal, however minuscule, a difference.

When dealing with gas molecules it becomes more of a philosophical question whether such difference is present or not.

Such a difference, nonetheless, suggests that there are two distinct ways to produce $\{1, 2\}$, whereas there is only a unique way to toss $\{1, 1\}$.

Quantitatively this supports the claim that $p_2 = 2p_1$.

Quantitatively this supports the claim that $p_2 = 2p_1$.

There are 6 probabilities with value p_1 and 15 with value $2p_1$.

Quantitatively this supports the claim that $p_2 = 2p_1$.

There are 6 probabilities with value p_1 and 15 with value $2p_1$.

The total is 1 so $p_1 = \frac{1}{36}$.

Quantitatively this supports the claim that $p_2 = 2p_1$.

There are 6 probabilities with value p_1 and 15 with value $2p_1$.

The total is 1 so $p_1 = \frac{1}{36}$.

The event $E =$ 'Toss 11 or higher' has probability $\frac{2}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$.

Quantitatively this supports the claim that $p_2 = 2p_1$.

There are 6 probabilities with value p_1 and 15 with value $2p_1$.

The total is 1 so $p_1 = \frac{1}{36}$.

The event $E =$ 'Toss 11 or higher' has probability $\frac{2}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$.

When gas-molecules are considered as one does in statistical mechanics, then it may well be questioned whether two distinct molecules are distinguishable at any level of resolution.

Quantitatively this supports the claim that $p_2 = 2p_1$.

There are 6 probabilities with value p_1 and 15 with value $2p_1$.

The total is 1 so $p_1 = \frac{1}{36}$.

The event $E =$ 'Toss 11 or higher' has probability $\frac{2}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$.

When gas-molecules are considered as one does in statistical mechanics, then it may well be questioned whether two distinct molecules are distinguishable at any level of resolution.

As one ponders this kind of issue more deeply there is a connection with the philosophical and religious concept of identity.

Even when it is agreed that two objects, say a pair of hydrogen atoms, cannot be distinguish experimentally, there is still a sense that each has its own identity.

Even when it is agreed that two objects, say a pair of hydrogen atoms, cannot be distinguish experimentally, there is still a sense that each has its own identity.

In the present setting one takes a pragmatic view and the sample space S is given a probability distribution that is not uniform.

Even when it is agreed that two objects, say a pair of hydrogen atoms, cannot be distinguish experimentally, there is still a sense that each has its own identity.

In the present setting one takes a pragmatic view and the sample space S is given a probability distribution that is not uniform.

An alternative is to assume at the outset that the two dice are distinguishable.

Even when it is agreed that two objects, say a pair of hydrogen atoms, cannot be distinguish experimentally, there is still a sense that each has its own identity.

In the present setting one takes a pragmatic view and the sample space S is given a probability distribution that is not uniform.

An alternative is to assume at the outset that the two dice are distinguishable.

Assume one is red and the other is green.

Even when it is agreed that two objects, say a pair of hydrogen atoms, cannot be distinguish experimentally, there is still a sense that each has its own identity.

In the present setting one takes a pragmatic view and the sample space S is given a probability distribution that is not uniform.

An alternative is to assume at the outset that the two dice are distinguishable.

Assume one is red and the other is green.

This time the sample space looks like

$$S' = \{(1,1), \ldots, (1,6), (2,1), \ldots, (2,6), \ldots, (6,1), \ldots, (6,6)\}.$$

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

Observe that $n(S') = 36$.

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

Observe that $n(S') = 36$.

The probability distribution is uniform, and each probability $p_1, \ldots, p_{36}$ is $\frac{1}{36}$.

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

Observe that $n(S') = 36$.

The probability distribution is uniform, and each probability $p_1, \ldots, p_{36}$ is $\frac{1}{36}$.

This time $E = \{(5, 6), (6, 5), (6, 6)\}$, and $n(E) = 3$, so $Pr(E) = \frac{3}{36} = \frac{1}{12}$.

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

Observe that $n(S') = 36$.

The probability distribution is uniform, and each probability $p_1, \ldots, p_{36}$ is $\frac{1}{36}$.

This time $E = \{(5, 6), (6, 5), (6, 6)\}$, and $n(E) = 3$, so $Pr(E) = \frac{3}{36} = \frac{1}{12}$.

This value is equal to the value calculated when the probability distribution is not uniform.

An outcome like $(2, 5)$ means specifically that the red die shows 2 and the green die shows 5.

Observe that $n(S') = 36$.

The probability distribution is uniform, and each probability $p_1, \ldots, p_{36}$ is $\frac{1}{36}$.

This time $E = \{(5, 6), (6, 5), (6, 6)\}$, and $n(E) = 3$, so $Pr(E) = \frac{3}{36} = \frac{1}{12}$.

This value is equal to the value calculated when the probability distribution is not uniform.

This vindicates the theory since the description of the event in this case only involves the random experiment and not the sample space employed.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

This also illustrates that in some cases it possible to select a sample space with uniform probability distribution although this is not necessarily suggested by the random experiment itself.

This also illustrates that in some cases it possible to select a sample space with uniform probability distribution although this is not necessarily suggested by the random experiment itself.

If the option is available to employ a sample space with uniform probability distribution, then by all means take it since it will simplify the calculations.

This also illustrates that in some cases it possible to select a sample space with uniform probability distribution although this is not necessarily suggested by the random experiment itself.

If the option is available to employ a sample space with uniform probability distribution, then by all means take it since it will simplify the calculations.

Suppose some event is known to take place, then it is possible to replace the original sample space with that event.

This also illustrates that in some cases it possible to select a sample space with uniform probability distribution although this is not necessarily suggested by the random experiment itself.

If the option is available to employ a sample space with uniform probability distribution, then by all means take it since it will simplify the calculations.

Suppose some event is known to take place, then it is possible to replace the original sample space with that event.

On the level of events one now only considers subsets of that event as the new events.

This also illustrates that in some cases it possible to select a sample space with uniform probability distribution although this is not necessarily suggested by the random experiment itself.

If the option is available to employ a sample space with uniform probability distribution, then by all means take it since it will simplify the calculations.

Suppose some event is known to take place, then it is possible to replace the original sample space with that event.

On the level of events one now only considers subsets of that event as the new events.

Events that were meaningful in the original sample space may still be meaningful, but only if they intersect the given event.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

This also illustrates that in some cases it possible to select a sample space with uniform probability distribution although this is not necessarily suggested by the random experiment itself.

If the option is available to employ a sample space with uniform probability distribution, then by all means take it since it will simplify the calculations.

Suppose some event is known to take place, then it is possible to replace the original sample space with that event.

On the level of events one now only considers subsets of that event as the new events.

Events that were meaningful in the original sample space may still be meaningful, but only if they intersect the given event.

The probabilities must be adjusted upwards to reflect that the new sample space is inside the old sample space.

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

For an event F the new probability is written $Pr(F|E)$ and it is given by

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}.$$

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

For an event F the new probability is written $Pr(F|E)$ and it is given by

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}.$$

This is known as the conditional probability, but it may be called the subset probability.

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

For an event F the new probability is written $Pr(F|E)$ and it is given by

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}.$$

This is known as the conditional probability, but it may be called the subset probability.

Observe that if $E = S$ then the formula becomes $Pr(F|S) = Pr(F)$.

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

For an event F the new probability is written $Pr(F|E)$ and it is given by

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}.$$

This is known as the conditional probability, but it may be called the subset probability.

Observe that if $E = S$ then the formula becomes $Pr(F|S) = Pr(F)$.

The formula is not symmetric and $Pr(E|F) = Pr(F|E)$ only if $Pr(E) = Pr(F)$.

If E is the event, then the adjustment is to multiply by $\frac{1}{Pr(E)}$.

For an event F the new probability is written $Pr(F|E)$ and it is given by

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)}.$$

This is known as the conditional probability, but it may be called the subset probability.

Observe that if $E = S$ then the formula becomes $Pr(F|S) = Pr(F)$.

The formula is not symmetric and $Pr(E|F) = Pr(F|E)$ only if $Pr(E) = Pr(F)$.

As expected, $Pr(E|E) = 1$.

The algebraic rewrite $Pr(F \cap E) = Pr(E)Pr(F|E)$ is used frequently.

The algebraic rewrite $Pr(F \cap E) = Pr(E)Pr(F|E)$ is used frequently.

Consider the unit square

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}.$$

The algebraic rewrite $Pr(F \cap E) = Pr(E)Pr(F|E)$ is used frequently.

Consider the unit square

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}.$$

Let S represent a sample space, and in the spirit of Venn diagrams let regions in the square represent events.

The algebraic rewrite $Pr(F \cap E) = Pr(E)Pr(F|E)$ is used frequently.

Consider the unit square

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}.$$

Let S represent a sample space, and in the spirit of Venn diagrams let regions in the square represent events.

The area of S is 1, and let the area of a region correspond to the probability of the corresponding event.

The algebraic rewrite $Pr(F \cap E) = Pr(E)Pr(F|E)$ is used frequently.

Consider the unit square

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}.$$

Let S represent a sample space, and in the spirit of Venn diagrams let regions in the square represent events.

The area of S is 1, and let the area of a region correspond to the probability of the corresponding event.

Let E be represented by the region left of the vertical line $x = 1/2$.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

The algebraic rewrite $Pr(F \cap E) = Pr(E)Pr(F|E)$ is used frequently.

Consider the unit square

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}.$$

Let S represent a sample space, and in the spirit of Venn diagrams let regions in the square represent events.

The area of S is 1, and let the area of a region correspond to the probability of the corresponding event.

Let E be represented by the region left of the vertical line $x = 1/2$.

Let F_α be represented by the region below the line from $(0, \alpha)$ to $(1, 1 - \alpha)$ where $0 \leq \alpha \leq 1$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$.

When $\alpha \leq 1/2$, then the rectangle has area $\alpha \times \frac{1}{2}$ and the triangle has area $(\frac{1}{2} - \alpha) \times \frac{1}{2} \times \frac{1}{2}$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$.

When $\alpha \leq 1/2$, then the rectangle has area $\alpha \times \frac{1}{2}$ and the triangle has area $(\frac{1}{2} - \alpha) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$, which is the same formula as before.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$.

When $\alpha \leq 1/2$, then the rectangle has area $\alpha \times \frac{1}{2}$ and the triangle has area $(\frac{1}{2} - \alpha) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$, which is the same formula as before.

In either case it is true that $Pr(F_\alpha | E) = \frac{2\alpha+1}{4}$.

Observe that $Pr(E) = Pr(F_\alpha) = 1/2$.

The region $F_\alpha \cap E$ may be split into a triangle and a rectangle.

Suppose $\alpha \geq 1/2$, then the rectangle has area $1/4$ and the triangle has area $(\alpha - \frac{1}{2}) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$.

When $\alpha \leq 1/2$, then the rectangle has area $\alpha \times \frac{1}{2}$ and the triangle has area $(\frac{1}{2} - \alpha) \times \frac{1}{2} \times \frac{1}{2}$.

It follows that $Pr(F_\alpha \cap E) = \frac{2\alpha+1}{8}$, which is the same formula as before.

In either case it is true that $Pr(F_\alpha|E) = \frac{2\alpha+1}{4}$.

When $\alpha = 0$ one gets $\frac{1}{4}$, so the probability of F_α has been reduced from $1/2$ to $1/4$.

When $\alpha = 1$ one gets $\frac{3}{4}$, so the probability of F_α has been increased from $1/2$ to $3/4$.

When $\alpha = 1$ one gets $\frac{3}{4}$, so the probability of F_α has been increased from $1/2$ to $3/4$.

In the case $\alpha = 1/2$, then the probability of F_α has not been changed at all.

When $\alpha = 1$ one gets $\frac{3}{4}$, so the probability of F_α has been increased from $1/2$ to $3/4$.

In the case $\alpha = 1/2$, then the probability of F_α has not been changed at all.

This last case is very exceptional but nonetheless of considerable interest.

When $\alpha = 1$ one gets $\frac{3}{4}$, so the probability of F_α has been increased from $1/2$ to $3/4$.

In the case $\alpha = 1/2$, then the probability of F_α has not been changed at all.

This last case is very exceptional but nonetheless of considerable interest.

In general, an event F is said to be independent of and event E if $Pr(F|E) = Pr(F)$.

When $\alpha = 1$ one gets $\frac{3}{4}$, so the probability of F_α has been increased from $1/2$ to $3/4$.

In the case $\alpha = 1/2$, then the probability of F_α has not been changed at all.

This last case is very exceptional but nonetheless of considerable interest.

In general, an event F is said to be independent of and event E if $Pr(F|E) = Pr(F)$.

Suppose F is independent of E, then

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{Pr(E)Pr(F|E)}{Pr(F)} = \frac{Pr(E)Pr(F)}{Pr(F)} = Pr(E)$$

It follows that E is independent of F, so the concept of independence is symmetric.

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

The sample space is $S = \{(r, r), (r, b), (b, r), (b, b)\}$.

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

The sample space is $S = \{(r, r), (r, b), (b, r), (b, b)\}$.

One may argue that $p_2 = 2p_1$, $p_3 = p_2$, and $p_4 = 2p_2$.

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

The sample space is $S = \{(r, r), (r, b), (b, r), (b, b)\}$.

One may argue that $p_2 = 2p_1$, $p_3 = p_2$, and $p_4 = 2p_2$.

It must be that $p_1 + 2p_1 + 2p_1 + 4p_1 = 1$ so $p_1 = 1/9$.

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

The sample space is $S = \{(r, r), (r, b), (b, r), (b, b)\}$.

One may argue that $p_2 = 2p_1$, $p_3 = p_2$, and $p_4 = 2p_2$.

It must be that $p_1 + 2p_1 + 2p_1 + 4p_1 = 1$ so $p_1 = 1/9$.

The event E has $Pr(E) = p_1 + p_2 = 3/9 = 1/3$.

It follows that E is independent of F, so the concept of independence is symmetric.

Consider the experiment of drawing a ball from an urn with 1 red ball and 2 blue balls and replacing it, and then drawing a ball again.

Let E be the event 'the first ball drawn is red', and F the event 'the second ball drawn is red'.

The sample space is $S = \{(r, r), (r, b), (b, r), (b, b)\}$.

One may argue that $p_2 = 2p_1$, $p_3 = p_2$, and $p_4 = 2p_2$.

It must be that $p_1 + 2p_1 + 2p_1 + 4p_1 = 1$ so $p_1 = 1/9$.

The event E has $Pr(E) = p_1 + p_2 = 3/9 = 1/3$.

The event F has $Pr(F) = p_1 + p_3 = 1/3$.

The event $F \cap E$ has $Pr(F \cap E) = 1/9$.

The event $F \cap E$ has $Pr(F \cap E) = 1/9$.

It follows that

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3} = Pr(F).$$

The event $F \cap E$ has $Pr(F \cap E) = 1/9$.

It follows that

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3} = Pr(F).$$

The events E and F are independent.

The event $F \cap E$ has $Pr(F \cap E) = 1/9$.

It follows that

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3} = Pr(F).$$

The events E and F are independent.

Observe that $Pr(E \cap F) = Pr(E)Pr(F)$ when the events are independent.

The event $F \cap E$ has $Pr(F \cap E) = 1/9$.

It follows that

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3} = Pr(F).$$

The events E and F are independent.

Observe that $Pr(E \cap F) = Pr(E)Pr(F)$ when the events are independent.

It is immediate that the events 'first ball drawn is blue' and 'second ball drawn is blue' are independent.

The event $F \cap E$ has $Pr(F \cap E) = 1/9$.

It follows that

$$Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3} = Pr(F).$$

The events E and F are independent.

Observe that $Pr(E \cap F) = Pr(E)Pr(F)$ when the events are independent.

It is immediate that the events 'first ball drawn is blue' and 'second ball drawn is blue' are independent.

In fact any pair of the four events are independent.

If this fact is known before one calculates, then $p_1 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, $p_2 = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, $p_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and

$$p_4 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}.$$

If this fact is known before one calculates, then $p_1 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, $p_2 = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, $p_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and

$$p_4 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}.$$

This calculation hints at the utility of knowing independence before calculating.

If this fact is known before one calculates, then $p_1 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, $p_2 = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, $p_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and

$$p_4 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}.$$

This calculation hints at the utility of knowing independence before calculating.

There is an intuitive understanding of the concept of independence that may be appealed to first.

If this fact is known before one calculates, then $p_1 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, $p_2 = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, $p_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and

$$p_4 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}.$$

This calculation hints at the utility of knowing independence before calculating.

There is an intuitive understanding of the concept of independence that may be appealed to first.

In the example one may argue that there is no way the outcome of the first draw influences the probabilities of the outcomes of the second draw.

If this fact is known before one calculates, then $p_1 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$, $p_2 = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, $p_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$, and

$$p_4 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}.$$

This calculation hints at the utility of knowing independence before calculating.

There is an intuitive understanding of the concept of independence that may be appealed to first.

In the example one may argue that there is no way the outcome of the first draw influences the probabilities of the outcomes of the second draw.

The trouble is that the simplifying impact on the calculations tempts one to assume that the events involved are independent.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

For instance, are the events 'mechanical failure of left wing jet engine' and 'mechanical failure of right wing jet engine' independent?

For instance, are the events 'mechanical failure of left wing jet engine' and 'mechanical failure of right wing jet engine' independent?

There may be a considerable amount of engineering design directed towards a desire to make the two events independent.

For instance, are the events 'mechanical failure of left wing jet engine' and 'mechanical failure of right wing jet engine' independent?

There may be a considerable amount of engineering design directed towards a desire to make the two events independent.

This by itself does not guarantee that this is indeed the case.

For instance, are the events 'mechanical failure of left wing jet engine' and 'mechanical failure of right wing jet engine' independent?

There may be a considerable amount of engineering design directed towards a desire to make the two events independent.

This by itself does not guarantee that this is indeed the case.

If the example is changed so that there is no replacement after the first draw, then the sample space is
$S = \{(r, b), (b, r), (b, b)\}$.

For instance, are the events 'mechanical failure of left wing jet engine' and 'mechanical failure of right wing jet engine' independent?

There may be a considerable amount of engineering design directed towards a desire to make the two events independent.

This by itself does not guarantee that this is indeed the case.

If the example is changed so that there is no replacement after the first draw, then the sample space is $S = \{(r, b), (b, r), (b, b)\}$.

This time $p_1 = p_2 = p_3 = 1/3$, $Pr(E) = Pr(F) = 2/3$, and $Pr(E \cap F) = 1/3$.

Now

$$Pr(F|E) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \neq Pr(F).$$

Now

$$Pr(F|E) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \neq Pr(F).$$

This technical dependence also agrees with the intuitive idea that the events here are not independent.

Now

$$Pr(F|E) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \neq Pr(F).$$

This technical dependence also agrees with the intuitive idea that the events here are not independent.

Do not confuse the notion of 'mutual exclusive events', which means $Pr(E \cup F) = Pr(E) + Pr(F)$, with 'independent events', which means $Pr(E \cap F) = Pr(E)Pr(F)$.

Now

$$Pr(F|E) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \neq Pr(F).$$

This technical dependence also agrees with the intuitive idea that the events here are not independent.

Do not confuse the notion of 'mutual exclusive events', which means $Pr(E \cup F) = Pr(E) + Pr(F)$, with 'independent events', which means $Pr(E \cap F) = Pr(E)Pr(F)$.

If E and F are independent, it is natural to ask if E^c and F^c are independent, or if E and F^c are independent.

First consider

$$Pr(F^c|E^c) = \frac{Pr(F^c \cap E^c)}{Pr(E^c)} = \frac{Pr((F \cup E)^c)}{Pr(E^c)} =$$

$$\frac{1 - Pr(E \cup F)}{Pr(E^c)} = \frac{1 - Pr(E) - Pr(F) + Pr(E \cap F)}{Pr(E^c)},$$

so

$$Pr(F^c|E^c) = \frac{1 - Pr(E) - Pr(F) + Pr(E)Pr(F)}{1 - Pr(E)} =$$

$$1 - Pr(F) = Pr(F^c).$$

First consider

$$Pr(F^c|E^c) = \frac{Pr(F^c \cap E^c)}{Pr(E^c)} = \frac{Pr((F \cup E)^c)}{Pr(E^c)} =$$

$$\frac{1 - Pr(E \cup F)}{Pr(E^c)} = \frac{1 - Pr(E) - Pr(F) + Pr(E \cap F)}{Pr(E^c)},$$

so

$$Pr(F^c|E^c) = \frac{1 - Pr(E) - Pr(F) + Pr(E)Pr(F)}{1 - Pr(E)} =$$

$$1 - Pr(F) = Pr(F^c).$$

Next consider

$$Pr(F^c|E) = \frac{Pr(F^c \cap E)}{Pr(E)} = \frac{Pr(E) - Pr(F \cap E)}{Pr(E)} =$$

$$\frac{Pr(E) - Pr(F)Pr(E)}{Pr(E)} = 1 - Pr(F) = Pr(F^c).$$

It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

They involve a nice mix of deMorgan's law, and other basic laws of set and probability theory.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

They involve a nice mix of deMorgan's law, and other basic laws of set and probability theory.

To organize calculations that involve conditional probabilities it is useful to draw a tree diagram.

It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

They involve a nice mix of deMorgan's law, and other basic laws of set and probability theory.

To organize calculations that involve conditional probabilities it is useful to draw a tree diagram.

For simplicity, consider the application where D is a given event, with P and N subsequent events.

It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

They involve a nice mix of deMorgan's law, and other basic laws of set and probability theory.

To organize calculations that involve conditional probabilities it is useful to draw a tree diagram.

For simplicity, consider the application where D is a given event, with P and N subsequent events.

The tree diagram has an initial node from where two edges emanate ending at a node marked D and a node marked D^c.

It is definitely worthwhile to scrutinize these two derivations to make sure that each step is understood.

They involve a nice mix of deMorgan's law, and other basic laws of set and probability theory.

To organize calculations that involve conditional probabilities it is useful to draw a tree diagram.

For simplicity, consider the application where D is a given event, with P and N subsequent events.

The tree diagram has an initial node from where two edges emanate ending at a node marked D and a node marked D^c.

Both node D and node D^c have a pair of edges emanating to their own pair of nodes marked $+$ and $-$, respectively.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

Think of D as 'having a particular disease', and p the probability of having the disease.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

Think of D as 'having a particular disease', and p the probability of having the disease.

Think of $+$ as 'testing positive' and $-$ as 'testing negative'.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

Think of D as 'having a particular disease', and p the probability of having the disease.

Think of $+$ as 'testing positive' and $-$ as 'testing negative'.

The q is known as the probability of a false negative.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

Think of D as 'having a particular disease', and p the probability of having the disease.

Think of $+$ as 'testing positive' and $-$ as 'testing negative'.

The q is known as the probability of a false negative.

The r is known as the probability of a false positive.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

Think of D as 'having a particular disease', and p the probability of having the disease.

Think of $+$ as 'testing positive' and $-$ as 'testing negative'.

The q is known as the probability of a false negative.

The r is known as the probability of a false positive.

In realistic applications all three numbers p, q, r are small.

To complete the tree we will add probabilities p and $1-p$ to the first pair of edges, in order.

Next add $1-q$ and q to the pair emanating from D, as well as r and $1-r$ from D^c, each, in order.

Think of D as 'having a particular disease', and p the probability of having the disease.

Think of $+$ as 'testing positive' and $-$ as 'testing negative'.

The q is known as the probability of a false negative.

The r is known as the probability of a false positive.

In realistic applications all three numbers p, q, r are small.

It is also reasonable to as reasonable to assume that tests are designed so that q is smaller than r.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

The following holds: $Pr(D) = p$, $Pr(D^c) = 1 - p$, $Pr(+|D) = 1 - q$, $Pr(-|D) = q$, $Pr(+|D^c) = r$, and $Pr(-|D^c) = 1 - r$.

The following holds: $Pr(D) = p$, $Pr(D^c) = 1 - p$, $Pr(+|D) = 1 - q$, $Pr(-|D) = q$, $Pr(+|D^c) = r$, and $Pr(-|D^c) = 1 - r$.

There are therefore 6 probabilities indicated in the tree diagram.

The following holds: $Pr(D) = p$, $Pr(D^c) = 1 - p$, $Pr(+|D) = 1 - q$, $Pr(-|D) = q$, $Pr(+|D^c) = r$, and $Pr(-|D^c) = 1 - r$.

There are therefore 6 probabilities indicated in the tree diagram.

The following 4 quantities are quickly calculated from data in the tree diagram: $Pr(D \cap +) = p(1 - q)$, $Pr(D \cap -) = pq$, $Pr(D^c \cap +) = (1 - p)r$, and $Pr(D^c \cap -) = (1 - p)(1 - r)$.

The following holds: $Pr(D) = p$, $Pr(D^c) = 1 - p$, $Pr(+|D) = 1 - q$, $Pr(-|D) = q$, $Pr(+|D^c) = r$, and $Pr(-|D^c) = 1 - r$.

There are therefore 6 probabilities indicated in the tree diagram.

The following 4 quantities are quickly calculated from data in the tree diagram: $Pr(D \cap +) = p(1 - q)$, $Pr(D \cap -) = pq$, $Pr(D^c \cap +) = (1 - p)r$, and $Pr(D^c \cap -) = (1 - p)(1 - r)$.

With a little bit more work one calculates
$Pr(+) = Pr(D \cap +) + Pr(D^c \cap +) = p(1 - q) + (1 - p)r$, and
$Pr(-) = Pr(D \cap -) + Pr(D^c \cap -) = pq + (1 - p)(1 - r)$.

Finally, there are 4 more additional quantities one may calculate:

$$Pr(D|+) = \frac{Pr(D \cap +)}{Pr(+)} = \frac{p(1-q)}{p(1-q)+(1-p)r},$$

$$Pr(D|-) = \frac{Pr(D \cap -)}{Pr(-)} = \frac{pq}{pq+(1-p)(1-r)},$$

$$Pr(D^c|+) = \frac{Pr(D^c \cap +)}{Pr(+)} = \frac{(1-p)r}{p(1-q)+(1-p)r},$$

$$Pr(D^c|-) = \frac{Pr(D^c \cap -)}{Pr(-)} = \frac{(1-p)(1-r)}{pq+(1-p)(1-r)}.$$

Finally, there are 4 more additional quantities one may calculate:

$$Pr(D|+) = \frac{Pr(D \cap +)}{Pr(+)} = \frac{p(1-q)}{p(1-q)+(1-p)r},$$

$$Pr(D|-) = \frac{Pr(D \cap -)}{Pr(-)} = \frac{pq}{pq+(1-p)(1-r)},$$

$$Pr(D^c|+) = \frac{Pr(D^c \cap +)}{Pr(+)} = \frac{(1-p)r}{p(1-q)+(1-p)r},$$

$$Pr(D^c|-) = \frac{Pr(D^c \cap -)}{Pr(-)} = \frac{(1-p)(1-r)}{pq+(1-p)(1-r)}.$$

In summary, there are 16 probabilities one may extract from the tree diagram with only some additional basic calculations.

Suppose $p = 2\%$, $q = 1\%$, and $r = 5\%$.

Suppose $p = 2\%$, $q = 1\%$, and $r = 5\%$.

The exact formula is

$$Pr(D|+) = \frac{2(100-1)}{2(100-1)+(100-2)5} = \frac{198}{198+490} = \frac{198}{688}.$$

Suppose $p = 2\%$, $q = 1\%$, and $r = 5\%$.

The exact formula is

$$Pr(D|+) = \frac{2(100-1)}{2(100-1)+(100-2)5} = \frac{198}{198+490} = \frac{198}{688}.$$

By ignoring products one gets the approximate formula

$$Pr(D|+) \approx \frac{p}{p+r} = \frac{2}{2+5} = \frac{2}{7}.$$

Suppose $p = 2\%$, $q = 1\%$, and $r = 5\%$.

The exact formula is

$$Pr(D|+) = \frac{2(100-1)}{2(100-1)+(100-2)5} = \frac{198}{198+490} = \frac{198}{688}.$$

By ignoring products one gets the approximate formula

$$Pr(D|+) \approx \frac{p}{p+r} = \frac{2}{2+5} = \frac{2}{7}.$$

The other approximate formulas are

$$Pr(D|-) \approx 0,$$

$$Pr(D^c|+) \approx \frac{r}{p+r},$$

$$Pr(D^c|-) \approx 1.$$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

The interest in these particular formulas is largely due to the fact that there is very little intuition regarding the values pf $P(D|+)$ and $P(D^c|+)$.

The interest in these particular formulas is largely due to the fact that there is very little intuition regarding the values pf $P(D|+)$ and $P(D^c|+)$.

Rewrite the approximate formulas one more time in the forms

$$Pr(D|+) \approx \frac{1}{1+\frac{r}{p}},$$

and

$$Pr(D^c|+) \approx \frac{1}{\frac{p}{r}+1}.$$

The interest in these particular formulas is largely due to the fact that there is very little intuition regarding the values pf $P(D|+)$ and $P(D^c|+)$.

Rewrite the approximate formulas one more time in the forms

$$Pr(D|+) \approx \frac{1}{1+\frac{r}{p}},$$

and

$$Pr(D^c|+) \approx \frac{1}{\frac{p}{r}+1}.$$

Observe that $Pr(D|+) + Pr(D^c|+) = 1$ both in the exact and the approximate case.

The interest in these particular formulas is largely due to the fact that there is very little intuition regarding the values pf $P(D|+)$ and $P(D^c|+)$.

Rewrite the approximate formulas one more time in the forms

$$Pr(D|+) \approx \frac{1}{1+\frac{r}{p}},$$

and

$$Pr(D^c|+) \approx \frac{1}{\frac{p}{r}+1}.$$

Observe that $Pr(D|+) + Pr(D^c|+) = 1$ both in the exact and the approximate case.

The approximate formulas supports the view that the crucial quantity in medical testing is the ratio of false positives to the likelihood of having the disease.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

This produces the odd circumstance where the societal concern that sick people spread the disease when given false negatives focuses the attention on q, whereas the individual's desire to assess the likelihood of having the disease focuses the attention on false positives and r.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

This produces the odd circumstance where the societal concern that sick people spread the disease when given false negatives focuses the attention on q, whereas the individual's desire to assess the likelihood of having the disease focuses the attention on false positives and r.

If the false positive probability is equal to the probability of having the disease, then both approximate values are 50%.

This produces the odd circumstance where the societal concern that sick people spread the disease when given false negatives focuses the attention on q, whereas the individual's desire to assess the likelihood of having the disease focuses the attention on false positives and r.

If the false positive probability is equal to the probability of having the disease, then both approximate values are 50%.

The exact formulas produce $\frac{1-q}{2-p-q}$ and $\frac{1-p}{2-p-q}$, so it is seen that this way of 'approximating' is not necessarily completely meaningful.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

The general tree diagram assumes the sample space has the partition $S = E_1 \cup \cdots \cup E_K$, where each pair $E_i, E_{i'}$ with $i \neq i'$ consists of mutually exclusive events.

The general tree diagram assumes the sample space has the partition $S = E_1 \cup \cdots \cup E_K$, where each pair $E_i, E_{i'}$ with $i \neq i'$ consists of mutually exclusive events.

It follows that there are K branches with a probability distribution $p_1, \ldots, p_K$ so that $p_1 + \cdots + p_K = 1$.

The general tree diagram assumes the sample space has the partition $S = E_1 \cup \cdots \cup E_K$, where each pair $E_i, E_{i'}$ with $i \neq i'$ consists of mutually exclusive events.

It follows that there are K branches with a probability distribution $p_1, \ldots, p_K$ so that $p_1 + \cdots + p_K = 1$.

At the second stage there are mutually exclusive events $F_1^i, \ldots, F_L^i$ and L_i branches emanating from each E_i.

The general tree diagram assumes the sample space has the partition $S = E_1 \cup \cdots \cup E_K$, where each pair $E_i, E_{i'}$ with $i \neq i'$ consists of mutually exclusive events.

It follows that there are K branches with a probability distribution $p_1, \ldots, p_K$ so that $p_1 + \cdots + p_K = 1$.

At the second stage there are mutually exclusive events $F_1^i, \ldots, F_L^i$ and L_i branches emanating from each E_i.

The reasoning illustrated in the case $K = L = 2$ generalizes, and the resulting formulas $Pr(E_i|F_j^i)$, sometimes referred to as the a priori probabilities, are given by the so-called Bayes Theorem.

When written in general form these formulas look intimidating, but if in specific case the procedure illustrated by the $K = L = 2$ case is employed, then these formulas follow naturally.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

What is the probability the customer gets at most 1 rotten egg?

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

What is the probability the customer gets at most 1 rotten egg?

(a) 3

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

What is the probability the customer gets at most 1 rotten egg?

(a) 3

(b) $\dfrac{C(97,12)}{C(100,12)}$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

What is the probability the customer gets at most 1 rotten egg?

(a) 3

(b) $\frac{C(97,12)}{C(100,12)}$

(c) $\frac{97}{100}$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

What is the probability the customer gets at most 1 rotten egg?

(a) 3

(b) $\frac{C(97,12)}{C(100,12)}$

(c) $\frac{97}{100}$

(d) $\frac{1}{12}$

A farmer has 100 eggs, but 3 are rotten. He selects a dozen at random for a customer.

What is the probability the customer gets at most 1 rotten egg?

(a) 3

(b) $\frac{C(97,12)}{C(100,12)}$

(c) $\frac{97}{100}$

(d) $\frac{1}{12}$

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

If E and F are events with $Pr(E) = .7$, $Pr(F) = .4$ and $Pr(E \bigcap F) = .3$, what is $Pr(E^c \bigcap F^c)$?

If E and F are events with $Pr(E) = .7$, $Pr(F) = .4$ and $Pr(E \bigcap F) = .3$, what is $Pr(E^c \bigcap F^c)$?

(a) .4

If E and F are events with $Pr(E) = .7$, $Pr(F) = .4$ and $Pr(E \bigcap F) = .3$, what is $Pr(E^c \bigcap F^c)$?

(a) .4

(b) .2

If E and F are events with $Pr(E) = .7$, $Pr(F) = .4$ and $Pr(E \bigcap F) = .3$, what is $Pr(E^c \bigcap F^c)$?

(a) .4

(b) .2

(c) .28

If E and F are events with $Pr(E) = .7$, $Pr(F) = .4$ and $Pr(E \bigcap F) = .3$, what is $Pr(E^c \bigcap F^c)$?

(a) .4

(b) .2

(c) .28

(d) .1

If E and F are events with $Pr(E) = .7$, $Pr(F) = .4$ and $Pr(E \bigcap F) = .3$, what is $Pr(E^c \bigcap F^c)$?

(a) .4

(b) .2

(c) .28

(d) .1

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

An exam has eight True/False questions. What is the probability that a student guessing will get at least one problem correct?

An exam has eight True/False questions. What is the probability that a student guessing will get at least one problem correct?

(a) $\frac{9}{2^8}$

An exam has eight True/False questions. What is the probability that a student guessing will get at least one problem correct?

(a) $\frac{9}{2^8}$

(b) $\frac{1}{8}$

An exam has eight True/False questions. What is the probability that a student guessing will get at least one problem correct?

(a) $\frac{9}{2^8}$

(b) $\frac{1}{8}$

(c) $\frac{1}{32}$

An exam has eight True/False questions. What is the probability that a student guessing will get at least one problem correct?

(a) $\frac{9}{2^8}$

(b) $\frac{1}{8}$

(c) $\frac{1}{32}$

(d) $\frac{255}{256}$

An exam has eight True/False questions. What is the probability that a student guessing will get at least one problem correct?

(a) $\frac{9}{2^8}$

(b) $\frac{1}{8}$

(c) $\frac{1}{32}$

(d) $\frac{255}{256}$

(e) None of the above.

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

Approximately, what is the probability that a randomly chosen person tests positive?

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

Approximately, what is the probability that a randomly chosen person tests positive?

(a) .01

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

Approximately, what is the probability that a randomly chosen person tests positive?

(a) .01

(b) .02

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

Approximately, what is the probability that a randomly chosen person tests positive?

(a) .01

(b) .02

(c) .3

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

Approximately, what is the probability that a randomly chosen person tests positive?

(a) .01

(b) .02

(c) .3

(d) .05

I develop a test for a disease that affects .01% of the population. If a person has the disease my test will be positive 80% of the time. If a person does not have the disease my test will be negative 70% of the time.

Approximately, what is the probability that a randomly chosen person tests positive?

(a) .01

(b) .02

(c) .3

(d) .05

(e) None of the above.

If you test positive, approximately what is the probability that you have the disease?

If you test positive, approximately what is the probability that you have the disease?

(a) .3

If you test positive, approximately what is the probability that you have the disease?

(a) .3

(b) .0005

If you test positive, approximately what is the probability that you have the disease?

(a) .3

(b) .0005

(c) .01

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

If you test positive, approximately what is the probability that you have the disease?

(a) .3

(b) .0005

(c) .01

(d) .0003

If you test positive, approximately what is the probability that you have the disease?

(a) .3

(b) .0005

(c) .01

(d) .0003

(e) .7

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

A pair of fair dice is cast. What is the probability that at least one four is cast?

A pair of fair dice is cast. What is the probability that at least one four is cast?

(a) $\frac{1}{2}$

A pair of fair dice is cast. What is the probability that at least one four is cast?

(a) $\frac{1}{2}$

(b) $\frac{1}{36}$

A pair of fair dice is cast. What is the probability that at least one four is cast?

(a) $\frac{1}{2}$

(b) $\frac{1}{36}$

(c) $\frac{1}{4}$

A pair of fair dice is cast. What is the probability that at least one four is cast?

(a) $\frac{1}{2}$

(b) $\frac{1}{36}$

(c) $\frac{1}{4}$

(d) $\frac{11}{25}$

A pair of fair dice is cast. What is the probability that at least one four is cast?

(a) $\frac{1}{2}$

(b) $\frac{1}{36}$

(c) $\frac{1}{4}$

(d) $\frac{11}{25}$

(e) None of the above.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Find $Pr(A \bigcup B)$

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Find $Pr(A \bigcup B)$

(a) 1.1

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Find $Pr(A \bigcup B)$

(a) 1.1

(b) .9

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Find $Pr(A \bigcup B)$

(a) 1.1

(b) .9

(c) .6

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Find $Pr(A \bigcup B)$

(a) 1.1

(b) .9

(c) .6

(d) .4

Let A and B be events in a sample space S such that:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(A \bigcap B) = .2$$

Find $Pr(A \bigcup B)$

(a) 1.1

(b) .9

(c) .6

(d) .4

(e) None of the above.

A card is picked from a deck. Let E be the event 'a club is picked' and F the event 'an ace is picked'. Which of the following is false?

A card is picked from a deck. Let E be the event 'a club is picked' and F the event 'an ace is picked'. Which of the following is false?

(a) $Pr(E) = \frac{1}{4}$

A card is picked from a deck. Let E be the event 'a club is picked' and F the event 'an ace is picked'. Which of the following is false?

(a) $Pr(E) = \frac{1}{4}$

(b) E and F are independent

A card is picked from a deck. Let E be the event 'a club is picked' and F the event 'an ace is picked'. Which of the following is false?

(a) $Pr(E) = \frac{1}{4}$

(b) E and F are independent

(c) E and F are mutually exclusive

A card is picked from a deck. Let E be the event 'a club is picked' and F the event 'an ace is picked'. Which of the following is false?

(a) $Pr(E) = \frac{1}{4}$

(b) E and F are independent

(c) E and F are mutually exclusive

(d) $Pr(F) = \frac{1}{13}$

A card is picked from a deck. Let E be the event 'a club is picked' and F the event 'an ace is picked'. Which of the following is false?

(a) $Pr(E) = \frac{1}{4}$

(b) E and F are independent

(c) E and F are mutually exclusive

(d) $Pr(F) = \frac{1}{13}$

(e) None of these.

A biased coin is flipped 3 times. If the coin comes up heads 60% of the time, what is the probability of getting 3 tails?

A biased coin is flipped 3 times. If the coin comes up heads 60% of the time, what is the probability of getting 3 tails?

(a) .216

A biased coin is flipped 3 times. If the coin comes up heads 60% of the time, what is the probability of getting 3 tails?

(a) .216

(b) .192

A biased coin is flipped 3 times. If the coin comes up heads 60% of the time, what is the probability of getting 3 tails?

(a) .216

(b) .192

(c) .064

A biased coin is flipped 3 times. If the coin comes up heads 60% of the time, what is the probability of getting 3 tails?

(a) .216

(b) .192

(c) .064

(d) .125

A biased coin is flipped 3 times. If the coin comes up heads 60% of the time, what is the probability of getting 3 tails?

(a) .216

(b) .192

(c) .064

(d) .125

(e) .4

If a fair coin is flipped 7 times, what is the probability of getting 4 heads and 3 tails?

If a fair coin is flipped 7 times, what is the probability of getting 4 heads and 3 tails?

(a) $\frac{35}{128}$

If a fair coin is flipped 7 times, what is the probability of getting 4 heads and 3 tails?

(a) $\frac{35}{128}$

(b) $\frac{1}{128}$

If a fair coin is flipped 7 times, what is the probability of getting 4 heads and 3 tails?

(a) $\frac{35}{128}$

(b) $\frac{1}{128}$

(c) $\frac{52}{128}$

If a fair coin is flipped 7 times, what is the probability of getting 4 heads and 3 tails?

(a) $\frac{35}{128}$

(b) $\frac{1}{128}$

(c) $\frac{52}{128}$

(d) $\frac{210}{128}$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

If a fair coin is flipped 7 times, what is the probability of getting 4 heads and 3 tails?

(a) $\frac{35}{128}$

(b) $\frac{1}{128}$

(c) $\frac{52}{128}$

(d) $\frac{210}{128}$

(e) $\frac{1}{16}$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

At a particular university, it is found that 60% of students that pass college algebra go on to pass calculus I. However, only 10% of those that do not pass college algebra go on to pass calculus I. If 55% pass college algebra, what percentage of students pass calculus I?

At a particular university, it is found that 60% of students that pass college algebra go on to pass calculus I. However, only 10% of those that do not pass college algebra go on to pass calculus I. If 55% pass college algebra, what percentage of students pass calculus I?

(a) 33%

At a particular university, it is found that 60% of students that pass college algebra go on to pass calculus I. However, only 10% of those that do not pass college algebra go on to pass calculus I. If 55% pass college algebra, what percentage of students pass calculus I?

(a) 33%

(b) 37.5%

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

At a particular university, it is found that 60% of students that pass college algebra go on to pass calculus I. However, only 10% of those that do not pass college algebra go on to pass calculus I. If 55% pass college algebra, what percentage of students pass calculus I?

(a) 33%

(b) 37.5%

(c) 54%

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

At a particular university, it is found that 60% of students that pass college algebra go on to pass calculus I. However, only 10% of those that do not pass college algebra go on to pass calculus I. If 55% pass college algebra, what percentage of students pass calculus I?

(a) 33%

(b) 37.5%

(c) 54%

(d) 23.5%

At a particular university, it is found that 60% of students that pass college algebra go on to pass calculus I. However, only 10% of those that do not pass college algebra go on to pass calculus I. If 55% pass college algebra, what percentage of students pass calculus I?

(a) 33%

(b) 37.5%

(c) 54%

(d) 23.5%

(e) 32.5%

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Suppose E and F are mutually exclusive events with $P(E) = .2$ and $P(F) = .5$. What is $P(E^c \cap F^c)$?

Suppose E and F are mutually exclusive events with $P(E) = .2$ and $P(F) = .5$. What is $P(E^c \cap F^c)$?

(a) .15

Suppose E and F are mutually exclusive events with $P(E) = .2$ and $P(F) = .5$. What is $P(E^c \cap F^c)$?

(a) .15

(b) 1

Suppose E and F are mutually exclusive events with $P(E) = .2$ and $P(F) = .5$. What is $P(E^c \cap F^c)$?

(a) .15

(b) 1

(c) .3

Suppose E and F are mutually exclusive events with $P(E) = .2$ and $P(F) = .5$. What is $P(E^c \cap F^c)$?

(a) .15

(b) 1

(c) .3

(d) .9

Suppose E and F are mutually exclusive events with $P(E) = .2$ and $P(F) = .5$. What is $P(E^c \cap F^c)$?

(a) .15

(b) 1

(c) .3

(d) .9

(e) .7

If one card is drawn from a 52 card deck, what is the probability it is an ace or a heart?

If one card is drawn from a 52 card deck, what is the probability it is an ace or a heart?

(a) $\frac{17}{52}$

If one card is drawn from a 52 card deck, what is the probability it is an ace or a heart?

(a) $\frac{17}{52}$

(b) $\frac{1}{4}$

If one card is drawn from a 52 card deck, what is the probability it is an ace or a heart?

(a) $\frac{17}{52}$

(b) $\frac{1}{4}$

(c) $\frac{1}{13}$

If one card is drawn from a 52 card deck, what is the probability it is an ace or a heart?

(a) $\frac{17}{52}$

(b) $\frac{1}{4}$

(c) $\frac{1}{13}$

(d) $\frac{5}{9}$

If one card is drawn from a 52 card deck, what is the probability it is an ace or a heart?

(a) $\frac{17}{52}$

(b) $\frac{1}{4}$

(c) $\frac{1}{13}$

(d) $\frac{5}{9}$

(e) $\frac{4}{13}$

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

If a multiple choice exam is given with 15 questions and 5 possible answers per question, what is the probability that random guessing will answer all equations incorrectly?

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

If a multiple choice exam is given with 15 questions and 5 possible answers per question, what is the probability that random guessing will answer all equations incorrectly?

(a) .0352

If a multiple choice exam is given with 15 questions and 5 possible answers per question, what is the probability that random guessing will answer all equations incorrectly?

(a) .0352

(b) $\frac{1}{3}$

If a multiple choice exam is given with 15 questions and 5 possible answers per question, what is the probability that random guessing will answer all equations incorrectly?

(a) .0352

(b) $\frac{1}{3}$

(c) 1

If a multiple choice exam is given with 15 questions and 5 possible answers per question, what is the probability that random guessing will answer all equations incorrectly?

(a) .0352

(b) $\frac{1}{3}$

(c) 1

(d) 3

If a multiple choice exam is given with 15 questions and 5 possible answers per question, what is the probability that random guessing will answer all equations incorrectly?

(a) .0352

(b) $\frac{1}{3}$

(c) 1

(d) 3

(e) .542

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Given two events, E and F, with $P(E) = .4$, $P(F|E) = .8$ and $P(F) = .7$, what is $P(E^c \cap F^c)$?

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

Given two events, E and F, with $P(E) = .4$, $P(F|E) = .8$ and $P(F) = .7$, what is $P(E^c \cap F^c)$?

(a) .22

Given two events, E and F, with $P(E) = .4$, $P(F|E) = .8$ and $P(F) = .7$, what is $P(E^c \cap F^c)$?

(a) .22

(b) .9

Given two events, E and F, with $P(E) = .4$, $P(F|E) = .8$ and $P(F) = .7$, what is $P(E^c \cap F^c)$?

(a) .22

(b) .9

(c) .46

Given two events, E and F, with $P(E) = .4$, $P(F|E) = .8$ and $P(F) = .7$, what is $P(E^c \cap F^c)$?

(a) .22

(b) .9

(c) .46

(d) .5

Given two events, E and F, with $P(E) = .4$, $P(F|E) = .8$ and $P(F) = .7$, what is $P(E^c \cap F^c)$?

(a) .22

(b) .9

(c) .46

(d) .5

(e) This situation is impossible.

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Given independent events, E and F, with $P(E) = .3$ and $P(F) = .4$, what is $P(F \cap E^c)$?

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

Given independent events, E and F, with $P(E) = .3$ and $P(F) = .4$, what is $P(F \cap E^c)$?

(a) .7

Given independent events, E and F, with $P(E) = .3$ and $P(F) = .4$, what is $P(F \cap E^c)$?

(a) .7

(b) .3

Given independent events, E and F, with $P(E) = .3$ and $P(F) = .4$, what is $P(F \cap E^c)$?

(a) .7

(b) .3

(c) .28

Given independent events, E and F, with $P(E) = .3$ and $P(F) = .4$, what is $P(F \cap E^c)$?

(a) .7

(b) .3

(c) .28

(d) .12

Given independent events, E and F, with $P(E) = .3$ and $P(F) = .4$, what is $P(F \cap E^c)$?

(a) .7

(b) .3

(c) .28

(d) .12

(e) .18

Suppose .1% of a population has John-Jacobs disease and a test is 99.5% accurate (meaning it is correct 99.5% of the time). If someone tests positive for John-Jacobs disease, what is the probability they actually have it?

Suppose .1% of a population has John-Jacobs disease and a test is 99.5% accurate (meaning it is correct 99.5% of the time). If someone tests positive for John-Jacobs disease, what is the probability they actually have it?

(a) .834

Suppose .1% of a population has John-Jacobs disease and a test is 99.5% accurate (meaning it is correct 99.5% of the time). If someone tests positive for John-Jacobs disease, what is the probability they actually have it?

(a) .834

(b) .995

Suppose .1% of a population has John-Jacobs disease and a test is 99.5% accurate (meaning it is correct 99.5% of the time). If someone tests positive for John-Jacobs disease, what is the probability they actually have it?

(a) .834

(b) .995

(c) .005

Suppose .1% of a population has John-Jacobs disease and a test is 99.5% accurate (meaning it is correct 99.5% of the time). If someone tests positive for John-Jacobs disease, what is the probability they actually have it?

(a) .834

(b) .995

(c) .005

(d) .462

Suppose .1% of a population has John-Jacobs disease and a test is 99.5% accurate (meaning it is correct 99.5% of the time). If someone tests positive for John-Jacobs disease, what is the probability they actually have it?

(a) .834

(b) .995

(c) .005

(d) .462

(e) .166

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation

Counting
Multiplication principle.
Counting

I choose 3 cards from a standard deck of 52 cards. What is the probability they are all red?

I choose 3 cards from a standard deck of 52 cards. What is the probability they are all red?

(a) .0577

I choose 3 cards from a standard deck of 52 cards. What is the probability they are all red?

(a) .0577

(b) .118

Set Theory
Finite sets.
Set notation.
Universal set and empty set.
Subsets.
Sets of sets.
Set-operations.
Examples.
Laws.
Commutative and associative.
DeMorgan's laws.
Proof I.
Proof II.
Venn-diagram I.
Venn-diagram II.
Exclusive OR.
Counting.
Counting subsets.
Notation
Counting
Multiplication principle.
Counting

I choose 3 cards from a standard deck of 52 cards. What is the probability they are all red?

(a) .0577

(b) .118

(c) .216

I choose 3 cards from a standard deck of 52 cards. What is the probability they are all red?

(a) .0577

(b) .118

(c) .216

(d) .125

I choose 3 cards from a standard deck of 52 cards. What is the probability they are all red?

(a) .0577

(b) .118

(c) .216

(d) .125

(e) .083