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Anders Linnér
alinner@math.niu.edu

Get in shape!

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Let $\xi : X \to Y$ be the operator that is defined by $\xi(x) \in Y$ such that $\xi(x)(t) = x'(t)$.

Consider the function $y \in Y$ given by $y(t) = \frac{1}{t}$.

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Is there any $x \in X$ such that $y = \xi(x)$?

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This separable first-order equation leads directly to the formula

$$x(t) = \int_1^t \frac{1}{u}\, du.$$

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To understand this formula one must understand the theory of integration leading up to the fundamental theorem of calculus, which in this particular case asserts that the derivative of the expression is in fact $\frac{1}{t}$.

Is it OK?

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It is crucial here that t is assumed to satisfy $t > 0$. With this assumption satisfied, and the fact that the function integrated is continuous on the domain of all positive numbers, means the basic calculus theory of Riemann integration suffices here.

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Once existence is established it is useful to introduce a name, and the most common choice is ln.

It follows that $\ln(1) = 0$ and $\ln'(t) = \frac{1}{t}$.

An interesting scalar field.

Let $\mathbb{R}$ be the real numbers and write $\mathbb{R}^2$ for $\mathbb{R} \times \mathbb{R}$.

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Let $\mathbb{R}$ be the real numbers and write $\mathbb{R}^2$ for $\mathbb{R} \times \mathbb{R}$.

Let $D = \mathbb{R}^2 - \{(0,0)\}$ and consider the scalar field $f : D \to \mathbb{R}$ given by

$$f(x,y) = \frac{1}{2}\ln(x^2 + y^2).$$

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This function is well-defined since $x^2 + y^2 > 0$ whenever $(x,y) \neq (0,0)$.

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To further analyze it, take the two partial derivatives

$$\frac{\partial f}{\partial x} = \frac{x}{x^2+y^2}, \quad \frac{\partial f}{\partial y} = \frac{y}{x^2+y^2}.$$

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$$\frac{\partial f}{\partial x} = \frac{x}{x^2+y^2}, \quad \frac{\partial f}{\partial y} = \frac{y}{x^2+y^2}.$$

This produces the so-called gradient vector field ∇f on D.

Higher derivatives.

Take one more partial derivative and get

$$\frac{\partial^2 f}{\partial x^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2},$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -\frac{2xy}{(x^2 + y^2)^2},$$

and

$$\frac{\partial^2 f}{\partial y^2} = \frac{x^2 - y^2}{(x^2 + y^2)^2}.$$

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If one adds the first and the last of the three formulas it is seen that f satisfies Laplace's equation, $\Delta f = 0$, so f is a so-called harmonic function.

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If one adds the first and the last of the three formulas it is seen that f satisfies Laplace's equation, $\Delta f = 0$, so f is a so-called harmonic function.

Another way to express this is via the divergence, so $\nabla \cdot \nabla f = 0$.

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Let $S^1 = f^{-1}(0)$ and parameterize it using $\gamma : [0, 2\pi] \to S^1$ given by $\gamma(t) = (\cos(t), \sin(t))$.

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This induces counter-clockwise orientation on S^1.

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Since on S^1 the gradient vector field is given by $\nabla f = (x, y)$ and the unit tangent vector field by $T = (-y, x)$, it follows that $\nabla f \cdot T = 0$, and therefore no work is performed by the vector field along S^1.

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Let $N = (x, y)$ be the 'outward' pointing unit normal vector.

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Let $N = (x, y)$ be the 'outward' pointing unit normal vector.

It follows that $\nabla f \cdot N = 1$ and the flux outwards through S^1 is 2π, the total length once around S^1.

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Anytime a vector field V is given by $V = \nabla f$, then V is said to have the potential f.

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In this case the work performed by V along a path P from p to q is given by

$$\int_P \nabla f \cdot T = [f]_p^q = f(q) - f(p).$$

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$$\int_P \nabla f \cdot T = [f]_p^q = f(q) - f(p).$$

Observe that this does not depend on the whole path, only the endpoints.

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Observe that this does not depend on the whole path, only the endpoints.

Moreover, if the path is closed so that $q = p$, then the total work is zero.

In the example it would therefore be true that the total work around S^1 is zero, which is in agreement with the previous reasoning.

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The '2-dimensional curl', which for $V = (P, Q)$ is given by

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y},$$

must vanish for a smooth vector field with potential.

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must vanish for a smooth vector field with potential.

The 2-dimensional version of Stokes' theorem,

$$\int_{\text{boundary of } D} V \cdot T = \iint_D (\nabla \times (P, Q, 0))_3,$$

will also produce zero work around S^1 when the vector field has a potential.

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Recall that

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Recall that

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As seen previously, the gradient vector field has outward flux 2π through S^1.

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Recall that

$$\int_{\text{boundary of } D} V \cdot N = \iint_D \nabla \cdot V.$$

As seen previously, the gradient vector field has outward flux 2π through S^1.

What is the source?

The divergence of the gradient vector field is zero everywhere on D!

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Consider the scalar field $g : \mathbb{R}^2 \to \mathbb{R}$ given by

$$g(x, y) = \frac{1}{2}(x^2 + y^2 + 1).$$

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Again the gradient vector field restricted to S^1 is given by (x, y).

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The work performed is always zero.

Flux without divergence?

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The outward flux through S^1 is again 2π.

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The outward flux through S^1 is again 2π.

The curl of the gradient vanishes again.

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Again the gradient vector field restricted to S^1 is given by (x, y).

The work performed is always zero.

The outward flux through S^1 is again 2π.

The curl of the gradient vanishes again.

So far there is no difference.

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When the divergence is calculated one sees that it is constant everywhere with value 2.

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The vector fields along S^1 are the same in the two cases.

Flux without divergence?

When the divergence is calculated one sees that it is constant everywhere with value 2.

When this is integrated over the region enclosed by S^1 one gets the value 2π, which is the flux.

The vector fields along S^1 are the same in the two cases.

The domains of the scalar fields f and g are different!

How do the domains differ?

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If one believes that $[0, 1)$ and $(0, 1)$ are isomorphic as sets via $\phi : [0, 1) \to (0, 1)$, then $\mathbb{R}^2$ and D are isomorphic as sets.

How do the domains differ?

If one believes that $[0, 1)$ and $(0, 1)$ are isomorphic as sets via $\phi : [0, 1) \to (0, 1)$, then $\mathbb{R}^2$ and D are isomorphic as sets.

To see this, use the isomorphism ϕ and extend it to a map from $[0, 1]$ to $(0, 1]$.

How do the domains differ?

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To see this, use the isomorphism ϕ and extend it to a map from $[0, 1]$ to $(0, 1]$.

Now use the identity on $\mathbb{R}^2 - [0, 1]$ to itself, and the extended map on the rest of the space.

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This is, however, not the issue here.

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The punctured plane has the restriction of the standard metric topology available in the plane.

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They differ as metric spaces as the punctured plane is not complete.

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The punctured plane has the restriction of the standard metric topology available in the plane.

They differ as metric spaces as the punctured plane is not complete.

The thing is that completeness is not a topological property.

How do the domains differ?

The critical hurdle is to figure out how to mathematically capture the presence of the ‘hole’.

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The critical hurdle is to figure out how to mathematically capture the presence of the ‘hole’.

This is where the fundamental group will become the key actor, and algebraic topology the general tool.

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As seen, adding a point to an infinite set may not change the view of the set.

Identify $\mathbb{R}^2$ with $\mathbb{C}$ and consider the map $\eta : [0, 1) \to \mathbb{C}$ given by $\eta(t) = e^{2\pi i t}$.

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Observe that $\eta([0, 1)) = S^1$.

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Identify $\mathbb{R}^2$ with $\mathbb{C}$ and consider the map $\eta : [0,1) \to \mathbb{C}$ given by $\eta(t) = e^{2\pi i t}$.

Observe that $\eta([0,1)) = S^1$.

This map is in fact a set-isomorphism from $[0,1)$ to S^1.

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Combining things one has that $(0, 1)$ and S^1 are set-isomorphic.

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Combining things one has that $(0,1)$ and S^1 are set-isomorphic.

Topology is supposed to distinguish between $(0,1)$ and S^1.

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Combining things one has that $(0,1)$ and S^1 are set-isomorphic.

Topology is supposed to distinguish between $(0,1)$ and S^1.

By the way, both spaces inherit a metric, $(0,1)$ from $\mathbb{R}$, and S^1 from $\mathbb{R}^2$.

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Combining things one has that $(0,1)$ and S^1 are set-isomorphic.

Topology is supposed to distinguish between $(0,1)$ and S^1.

By the way, both spaces inherit a metric, $(0,1)$ from $\mathbb{R}$, and S^1 from $\mathbb{R}^2$.

Anything true about all metric space will therefore not be useful when searching for a difference between the two spaces.

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Let S^2 be the set of points

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\},$$

the so-called unit-sphere.

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The straight line that emanates from $(0, 0, 1)$ and passes through a point $p \in S^2$ intersects the xy-coordinate plane at some point $q = (x, y, 0)$.

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The straight line that emanates from $(0, 0, 1)$ and passes through a point $p \in S^2$ intersects the xy-coordinate plane at some point $q = (x, y, 0)$.

This map, stereo-graphic projection, produces a set-isomorphism from the punctured sphere to $\mathbb{R}^2$.

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There is another set-isomorphism from $\mathbb{R}^2$ to the punctured plane.

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There is another set-isomorphism from $\mathbb{R}^2$ to the punctured plane.

Now send $(0, 0, 1)$ to the point missing from $\mathbb{R}^2$.

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S^2 inherits a metric from $\mathbb{R}^3$.

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S^2 inherits a metric from $\mathbb{R}^3$.

Our intuition suggests that $(0, 1)$ does not have a 'hole', whereas S^1 does.

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Now send $(0, 0, 1)$ to the point missing from $\mathbb{R}^2$.

This produces a set-isomorphism from S^2 to $\mathbb{R}^2$.

S^2 inherits a metric from $\mathbb{R}^3$.

Our intuition suggests that $(0, 1)$ does not have a 'hole', whereas S^1 does.

Similarly, $\mathbb{R}^2$ does not have a 'hole', whereas S^2 does.

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What do we mean by a hole?

What about $S^1 \times \mathbb{R}$, $S^1 \times S^1$, $S^2 \times \mathbb{R}$, $S^2 \times S^1$, or $S^2 \times S^2$?

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What about $S^1 \times \mathbb{R}$, $S^1 \times S^1$, $S^2 \times \mathbb{R}$, $S^2 \times S^1$, or $S^2 \times S^2$?

Do they have holes, if so, of what kind?

A closer look.

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Let $\epsilon > 0$ be given and consider the domain

$$A_\epsilon = \left\{(x, y) \in \mathbb{R}^2 \mid \epsilon \leq x^2 + y^2 \leq 1\right\}.$$

What is the outward flux from A_ϵ?

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$$A_\epsilon = \left\{(x, y) \in \mathbb{R}^2 \mid \epsilon \leq x^2 + y^2 \leq 1\right\}.$$

What is the outward flux from A_ϵ?

It is now necessary to add the integral $\nabla f \cdot N$ along $C_\epsilon = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = \epsilon\}$ to the previous value.

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This time one gets $\frac{1}{\epsilon}(x, y) \cdot \frac{1}{\sqrt{\epsilon}}(-x, -y) = -\frac{1}{\sqrt{\epsilon}}$.

A closer look.

Let $\epsilon > 0$ be given and consider the domain

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So for any $\epsilon > 0$ the new integral has value -2π.

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So for any $\epsilon > 0$ the new integral has value -2π.

The total outward flux is therefore zero, which is in agreement with Gauss.

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The total outward flux is therefore zero, which is in agreement with Gauss.

The collection of points on and inside S^1 differ from A_ϵ for each $\epsilon > 0$.

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Now consider the scalar field g.

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Now consider the scalar field g.

This time one gets $(x, y) \cdot \frac{1}{\sqrt{\epsilon}}(-x, -y) = -\sqrt{\epsilon}$.

A closer look.

Now consider the scalar field g.

This time one gets $(x, y) \cdot \frac{1}{\sqrt{\epsilon}}(-x, -y) = -\sqrt{\epsilon}$.

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This time one gets $(x, y) \cdot \frac{1}{\sqrt{\epsilon}}(-x, -y) = -\sqrt{\epsilon}$.

So for any $\epsilon > 0$ the new integral has value $-2\pi\epsilon$.

The total outward flux is therefore $2\pi - 2\pi\epsilon$, which is in agreement with Gauss since the divergence is 2 and $2 \cdot (\pi(1)^2) - 2(\pi \cdot (\sqrt{\epsilon})^2) = 2\pi - 2\pi\epsilon$.

A closer look.

Now consider the scalar field g.

This time one gets $(x, y) \cdot \frac{1}{\sqrt{\epsilon}}(-x, -y) = -\sqrt{\epsilon}$.

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The total outward flux is therefore $2\pi - 2\pi\epsilon$, which is in agreement with Gauss since the divergence is 2 and $2 \cdot (\pi(1)^2) - 2(\pi \cdot (\sqrt{\epsilon})^2) = 2\pi - 2\pi\epsilon$.

In this case the flux through C_ϵ tends to zero as ϵ tends to zero.

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Suppose $a, b \in \mathbb{R}$ satisfy $a < b$.

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Suppose $a, b \in \mathbb{R}$ satisfy $a < b$.

Write

$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}.$$

Open intervals.

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Suppose $a, b \in \mathbb{R}$ satisfy $a < b$.

Write

$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}.$$

The context will make sure this is not confused with the notation for a point in $\mathbb{R}^2$.

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A finite intersection of open intervals is either empty or an open interval.

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Unions are not well-behaved.

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The union of $(0, 1)$ and $(2, 3)$ is not an open interval.

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The idea is to localize things.

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A subset $U \subset \mathbb{R}$ is said to be open if to each $x \in U$ there is some $r_x > 0$ such that $(x - r_x, x + r_x) \subset U$.

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It is an important technical exercise to establish that an open interval is indeed open.

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Now any union of open sets in $\mathbb{R}$ is open.

While finite intersection of open sets in $\mathbb{R}$ are open,

$$\bigcap_{k=1}^{\infty}(-\frac{1}{k},\frac{1}{k})=\{0\}$$

is not an open set.

Open sets in $\mathbb{R}$.

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Observe that it is important to not drop the braces.

Open sets in metric spaces.

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Consider a metric space (X, d) where d is the metric.

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For $r > 0$ let $B_x(r)$ be given by

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For $r > 0$ let $B_x(r)$ be given by

$$B_x(r) = \{y \in X \mid d(y, x) < r\}.$$

It is known as the open ball of radius r centered at x,

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(1) It is immediate that X and $\emptyset$ are open.

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The collection of all open sets in X is known as the 'metric topology' of (X, d).

Topology.

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There may be other collections τ of subsets in X that satisfy (1)-(3).

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A simple example is given by the collection of all subsets, which is known as the 'discrete topology'.

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Another 'extreme' example is given by the collection $\{\emptyset, X\}$.

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This topology is known as the 'indiscrete topology'.

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An interesting question to contemplate is whether it is possible to find a metric that produces a given topology.

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In the discrete case the answer is yes.

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In the indiscrete case the answer is no.

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The discrete and indiscrete topology is available in any set, whether that set has a metric defined on it or not.

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Topologies on finite sets are known as finite topologies.

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There is no metric on this X that produces Sierpiński's topology.

Essentially the same topologies.

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Now let $\tau_0 = \{\emptyset, \{b\}, X\}$.

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Now let $\tau_0 = \{\emptyset, \{b\}, X\}$.

This is also a topology, and it is 'essentially' the same as τ.

Essentially the same topologies.

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Now let $\tau_0 = \{\emptyset, \{b\}, X\}$.

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To make this precise consider the map given by $\phi(a) = b$ and $\phi(b) = a$.

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But more is true, it takes open sets to open sets and the inverse does too.

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A map with these properties is said to be a homeomorphism from (X, τ) to (X, τ_0).

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The two spaces involved are said to be homeomorphic.

Essentially the same topologies.

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The two spaces involved are said to be homeomorphic.

Much of topology concerns settling whether two given spaces are homeomorphic or not.

Homeomorphisms.

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It is in general not trivial to produce a set-isomorphism so it should come as no surprise that it is even more difficult to produce a homeomorphism.

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To rule out the existence of a homeomorphism when the spaces are not homeomorphic also seems like a daunting task.

Homeomorphisms.

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To rule out the existence of a homeomorphism when the spaces are not homeomorphic also seems like a daunting task.

An idea that will be helpful is to search for properties that must be shared by homeomorphic spaces.

Homeomorphisms.

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This suggests that $(0, 1)$ and S^1 are not homeomorphic.

Closed sets.

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A topology specifies which subsets are considered open in addition to the empty set and the space itself.

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The presence of some subset other than the empty set and the whole space that is both open and closed is a topological property.

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The presence of some subset other than the empty set and the whole space that is both open and closed is a topological property.

The absence of such a set in any indiscrete topology and the presence of many such sets in any discrete topology confirms the suspicion that no indiscrete and discrete topologies are homeomorphic.

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$$\bigcup_{k=2}^{\infty} [0, 1 - \frac{1}{k}] = [0, 1)$$

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Subsets with a single point are known as singletons.

The indiscrete and Sierpiński's topologies show that singletons need not be closed.

Counting finite topologies.

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To get started let $X = \{a, b\}$.

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There is a total of $2^2 = 4$ subsets available.

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It turns out the all the 4 possibilities produce a topology.

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In summary one has the indiscrete, two Sierpiński topologies, and the discrete topology.

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In the collection of all topologies available in a space define two topologies to be equivalent whenever they are homeomorphic.

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This produces an equivalence relation and the count that really is of interest is the number of equivalence classes.

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In the present case this count is 3.

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A little bit of work remains to establish that the indiscrete topology is not homeomorphic to Sierpiński's topology.

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There is a total of $2^3 = 8$ subsets available.

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Now consider $X = \{a, b, c\}$.

There is a total of $2^3 = 8$ subsets available.

This means that this time there are 6 choices with $2^6 = 64$ possible decisions.

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It turns out that out of the remaining 62 there are 27 that are in fact topologies,

Counting finite topologies.

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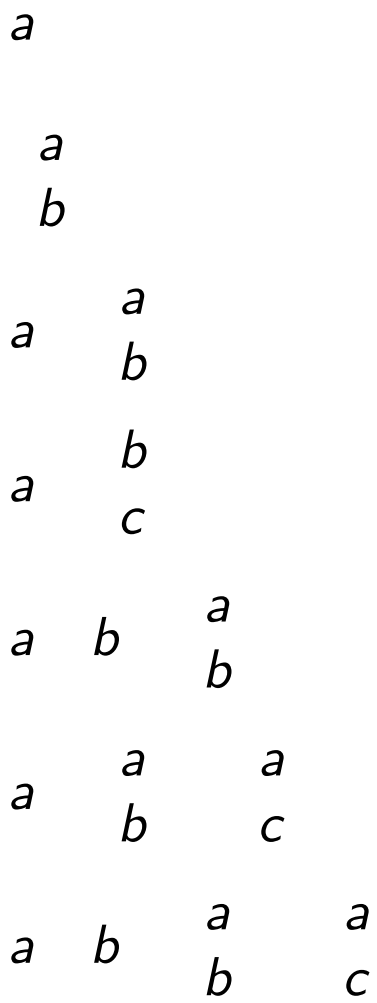
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Other than the indiscrete and the discrete they are represented by

Essentially different 3-point topologies.

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What happens when the number of points increases?

The general sequence N_k, where k is the number of points, may be written as $N_2 = 3$, $N_3 = 9$, $N_4 = 33$, $N_5 = 139$, $N_6 = 718$, $N_7 = 4535$, $N_8 = 35979$, $N_9 = 363083$, $N_{10} = 4717687$, $N_{11} = 79501654$, $N_{12} = 1744252509$, $N_{13} = 49872339897$, $N_{14} = 1856792610995$, $N_{15} = 89847422244493$, and $N_{16} = 5637294117525695$.

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Apparently there is a vast number of topologies to ponder.

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Almost all topologies that appear in practice are related to some other mathematical structure.

Separation.

If a point x belongs to an open set U, then U is said to be a neighborhood of x, or more precisely, an open neighborhood of x.

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A pair of points $x, y \in X$ are said to be topologically distinguishable if there is an open set that contains one of the points but not the other.

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A pair of points $x, y \in X$ are said to be topologically distinguishable if there is an open set that contains one of the points but not the other.

There is no pair of topologically distinguishable points in the indiscrete topology.

Separation.

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Sierpiński's topology has a pair of topologically distinguishable points so it is therefore not homeomorphic to the indiscrete topology.

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If any pair of distinct points in a space are topologically distinguishable, then the space is said to be Kolmogorov, or T_0.

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A space (X, τ) is Hausdorff, or T_2, if any distinct pair of points $x, y \in X$ has a pair of open sets $U_x, V_y \in \tau$ with $U_x \cap V_y = \emptyset$.

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$$B_x(\frac{1}{2}d(x,y)) \cap B_y(\frac{1}{2}d(x,y)) = \emptyset.$$

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It follows that metric spaces are Hausdorff.

Separation.

After some thought it is possible to conclude that if all singletons are closed in a finite space, then the space must be T_2.

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One idea that is pursued is to replace individual points by closed sets.

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One idea that is pursued is to replace individual points by closed sets.

This introduces a degree of uncertainty since singletons need not be closed.

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A space that, like the indiscrete topology, has no subset other than the empty set and the whole space that is both open and closed is said to be connected.

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Consider the sets $A = \{x \in \mathbb{R} \mid x \in U \text{ and } x < a\}$ and $B = \{x \in \mathbb{R} \mid x \in U \text{ and } x > a\}$.

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Since A is bounded above by a, there is a least upper bound α.

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If $\alpha \in U^c$, then $(\alpha - \varepsilon, \alpha + \varepsilon) \subset U^c$ for some $\varepsilon > 0$ because U^c is open.

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Let β be the greatest lower bound of B.

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It follows that no such U exists and hence $\mathbb{R}$ is connected.

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Variations of this reasoning prove that rays and intervals, whether open, closed, or mixed, are in fact connected.

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A connected set in $\mathbb{R}$ must in fact be of this kind.

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Variations of this reasoning prove that rays and intervals, whether open, closed, or mixed, are in fact connected.

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The concept of a space being connected does not coexist well with the set operations.

Unions, intersections and complements will not in general continue to be connected when the spaces involved are connected.

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Suppose $A \subset (X, \tau)$ is given and consider the nonempty collection

$$\{B \subset X \mid B^c \in \tau,\, A \subset B\}\,.$$

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One may also write A^- for the closure.

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Observe that $A^{--} = A^-$, and if $C \subset X$ is a closed set, then $C^- = C$.

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Every subset of a topological space is contained in a closed set, namely the space itself.

Suppose $A \subset (X, \tau)$ is given and consider the nonempty collection

$$\{B \subset X \mid B^c \in \tau,\, A \subset B\}\,.$$

The intersection of this family's members is a closed set that contains A and is denoted by $\bar{A}$.

One may also write A^- for the closure.

Observe that $A^{--} = A^-$, and if $C \subset X$ is a closed set, then $C^- = C$.

The set $\partial A = A^- \cap A^{c-}$ is a closed set known as the **topological** boundary of A.

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If $A \subset X$ satisfies $A^- = X$, then A is said to be dense in X.

Interiors.

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Observe that $A^{\circ\circ} = A^\circ$, and if $C \in \tau$, then $C^\circ = C$.

Given any $A \subset (X, \tau)$ the space X partitions as

$$X = A^\circ \cup \partial A \cup A^{c\circ}.$$

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Observe that $A^{\circ\circ} = A^\circ$, and if $C \in \tau$, then $C^\circ = C$.

Given any $A \subset (X, \tau)$ the space X partitions as

$$X = A^\circ \cup \partial A \cup A^{c\circ}.$$

Note that $\mathbb{Q}^\circ = \emptyset$, $\mathbb{Q}^- = \mathbb{R}$, and $\partial\mathbb{Q} = \mathbb{R}$.

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Let (X, τ) be a topological space and A a subset of X.

The collection of subsets $U \subset A$ such that $U = A \cap V$, where $V \in \tau$, is in fact a topology in A.

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When there is a chain $A \subset B \subset X$, then write τ_B for the subspace topology on B induced from X and similarly τ_A for the one induced on A by X.

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It is a technical exercise to verify that $\tau_A = \tau_A^B$.

Connected sets and closures.

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If $A \subset (X, \tau)$ is connected and dense in X, then X is connected.

Connected sets and closures.

If $A \subset (X, \tau)$ is connected and dense in X, then X is connected.

To see this, assume U is a nontrivial open and closed set in X.

Connected sets and closures.

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Since A is dense, neither of these two sets are empty.

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Since A is dense, neither of these two sets are empty.

Moreover, if $U \cap A = A$, then $A \subset U$, which again is incompatible with A being dense.

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Moreover, if $U \cap A = A$, then $A \subset U$, which again is incompatible with A being dense.

Similarly, $U^c \cap A = A$ is also impossible.

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To see this, assume U is a nontrivial open and closed set in X.

It follows that $U \cap A$ and $U^c \cap A$ are open in the subspace topology of A.

Since A is dense, neither of these two sets are empty.

Moreover, if $U \cap A = A$, then $A \subset U$, which again is incompatible with A being dense.

Similarly, $U^c \cap A = A$ is also impossible.

It follows that $U \cap A$ is both open and closed in the subspace topology of A, and hence A is not connected.

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The 'closure operation' does not 'rip a space apart'.

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Let (X, τ) be a topological space and write τ^c for the collection of closed sets.

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Let (X, τ) be a topological space and write τ^c for the collection of closed sets.

Denote the collection of all subsets of X by $P(X)$.

Connected sets and closures.

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Let (X, τ) be a topological space and write τ^c for the collection of closed sets.

Denote the collection of all subsets of X by $P(X)$.

Let $\phi : P(X) \to P(X)$ be given $\phi(A) = A^c$.

Connected sets and closures.

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One may consider the restricted maps $\phi : \tau \to \tau^c$ and $\phi : \tau^c \to \tau$.

Closures and complements.

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Observe that $\varphi(P(X)) = \tau^{c}$.

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Consider the map $\varphi : P(X) \to P(X)$ given by $\varphi(A) = A^{-}$.

Observe that $\varphi(P(X)) = \tau^{c}$.

It is also true that $\phi \circ \phi(A) = A$ and $\varphi \circ \varphi(A) = \varphi(A)$.

Closures and complements.

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It is reasonable to ask what may be said about $\varphi \circ \phi$, $\phi \circ \varphi$, or $\phi \circ \varphi \circ \phi$, $\varphi \circ \phi \circ \varphi$, and the analogous continuations.

Closures and complements.

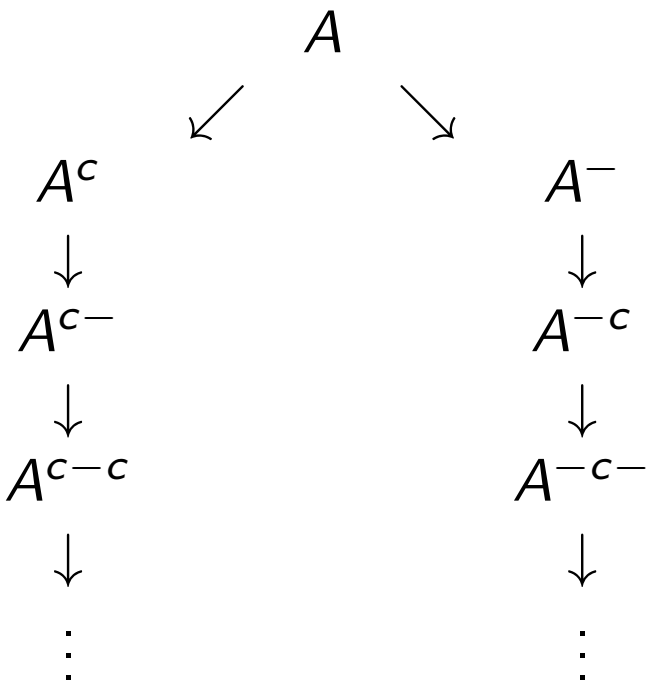
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Specifically, what may said about the following tree?


A
A^c
A^-
A^{c-}
A^{-c}
A^{c-c}
A^{-c-}

Compact spaces.

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Whereas the concepts of separation and a connected space involve fairly immediate geometric ideas, the concept of a compact space does not.

Compact spaces.

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The term compact may not capture the essence of the underlying idea, and words like confined or inescapable would do a better job.

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Suppose (X, τ) is a topological space, and let $U_\lambda \in \tau$ be a collection of subsets of X indexed by λ.

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The collection is said to be an open cover of X if its union is all of X.

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The space X is said to be compact if from any open cover it is possible to extract a finite number of members that still form an open cover.

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The extracted open sets are known as a finite subcover of the original open cover.

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The collection

$$\left\{(\frac{1}{k},1)\right\}_{k=2}^{\infty}$$

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Each finite topological space is compact since each open cover must be finite.

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More generally, a topological space with only a finite number of open sets must be compact.

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Compact spaces.

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Let A be the set $a \in [0, 1]$ such that $[0, a]$ is contained in a finite number of the U_λ.

Since some U_λ contains 0, it must be that $A \neq \emptyset$.

Compact spaces.

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Let A be the set $a \in [0, 1]$ such that $[0, a]$ is contained in a finite number of the U_λ.

Since some U_λ contains 0, it must be that $A \neq \emptyset$.

It is also true that A is bounded above by 1.

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Let A be the set $a \in [0, 1]$ such that $[0, a]$ is contained in a finite number of the U_λ.

Since some U_λ contains 0, it must be that $A \neq \emptyset$.

It is also true that A is bounded above by 1.

Let α be the least upper bound of A.

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The U_{λ_0} that contains α may be added to A so that $[0, a]$ is covered by a finite number of open sets the open cover where $a > \alpha$, which is impossible.

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Conclude that $\alpha = 1$ is the only possibility.

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Add U_{λ_1} to the finite open cover of A.

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There is a U_{λ_1} in the original open cover such that $1 \in U_{\lambda_1}$. Add U_{λ_1} to the finite open cover of A.

There is no $x \in [0, 1)$ that is neither in U_{λ_1} nor in A because such an x would be an upper bound less than α.

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In $\mathbb{R}^n$ it is well-known that a subset is compact if and only if it is closed and bounded.

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The issue that comes up in general is whether when $A \subset X$ is compact it is also closed as a subset in X.

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Since every non-trivial subset of a space with the indiscrete topology is compact and none is closed, the answer in general is no.

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In this last case it is not at all assumed that f is invertible.

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If f is invertible, and both f and f^{-1} are open, then f is a homeomorphism,

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If f is invertible, and both f and f^{-1} are open, then f is a homeomorphism,

This amounts to the fact that both f and f^{-1} are continuous.

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If to each pair $p, q \in X$ there is a path $\gamma_{p,q}$ such that $\gamma_{p,q}(0) = p$ and $\gamma_{p,q}(1) = q$, then X is said to be path-connected.

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Path-connected spaces are connected.

Suppose $f : X \to Y$ is continuous and X compact.

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To see this, let $y \in f(X)$ so that $y = f(x)$.

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Let $U = f^{-1}(V)$ and observe that U is nontrivial, and also open by the continuity of f.

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Now $f^{-1}(W)$ is open in X.

The claim is that $X - U = f^{-1}(W)$.

Suppose $x \in X - U$ and observe that $f(x) \in f(X) - V$ so $f(x) \in W$.

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It follows that $f(x) \notin V$ and hence $x \notin U$.

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One implication of this result is seen in Calculus and the intermediate value theorem.

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Observe that the image of a path is always both connected and compact.

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There are disturbing examples of so-called space-filling curves where, for instance, $f([0,1]) = [0,1] \times [0,1]$.

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There are disturbing examples of so-called space-filling curves where, for instance, $f([0,1]) = [0,1] \times [0,1]$.

Knowing such an example proves that $[0,1] \times [0,1]$ is compact, which is a nontrivial result.

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The space

$$Z = \left\{ (t, \sin(\frac{\pi}{t})) \mid t \in (0, 1] \right\} \subset \mathbb{R}^2$$

is path-connected, and hence connected.

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$$Z = \left\{ (t, \sin(\frac{\pi}{t})) \mid t \in (0, 1] \right\} \subset \mathbb{R}^2$$

is path-connected, and hence connected.

Let $Y = \{(0, y) \mid -1 \leq y \leq 1\}$ and observe that $X = Z^- = Y \cup Z$ is connected.

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It is very demanding technically to prove that X is not path-connected.

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One of the most important tasks in all of mathematics is to establish conditions on X and f that guarantee that there is some $x \in X$ such that $f(x) = \alpha$.

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The example $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^x$ shows that $x_k = -k$ has $f(x_k)$ tending to $\alpha = 0$, whereas the x_k do not tend to any real number.

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A directed set D is a set with a preorder $\preceq$ such that to each pair $x, y \in D$ there is some $z \in D$ such that $x \preceq z$ and $y \preceq z$.

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Set-inclusion, $\preceq = \subset$, and reverse set-inclusion, $\preceq = \supset$, offer two more useful examples.

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The most basic example is given by $x : \mathbb{N} \to X$.

In this case x is called a sequence in X, and one writes $x(k) = x_k$.

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If to each $\lambda_0 \in D$ there is a different $\lambda_1 \in D$ such that $\lambda_0 \preceq \lambda_1$ and $x_{\lambda_1} \in U$, then the net x_λ is said to be frequently in U.

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In Hausdorff spaces a limit is unique.

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In any topological space (X, τ), $U \subset X$ is open if and only if no net in $X - U$ has a limit in U.

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Proof: Given $U \in \tau$ let $x_\lambda \in X - U$.

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Conversely, suppose $A \subset X$ is not open and select $x_\infty \in A$ carefully so that each $U \in \tau$ with $x_\infty \in U$ has $U \cap A^c \neq \emptyset$.

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This produces a net x_U where the directed set is ordered by reverse inclusion.

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Observe that if $U \preceq V$ then $V \subset U$ and $x_V \in U$ so $x_U \to x_\infty$ holds.

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Let U be open with $x_\infty \in U$. There is a λ_U such that $\lambda_U \preceq \lambda$ then $x_\lambda \in U$.

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In particular, $x_{\lambda_U} \in U$.

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In particular, $x_{\lambda_U} \in U$.

It follows that for each $U \in \tau$ with $x_\infty \in U$ it must be that $U \cap A \neq \emptyset$, and hence $x_\infty \in A^-$.

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For each U in the collection it is true that there is some $x_U \in A \cap U$ since $x_\infty \in A^-$.

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The claim now is that $x_U \to x_\infty$.

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Suppose $V \in \tau$ has $x_\infty \in V$.

Now if $V \preceq U$ then $U \subset V$ and $x_U \in V$.

This means $x_U \to x_\infty$.

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Suppose $f : X \to Y$, then f is continuous if and only if each convergent net $x_\lambda \to x_\infty$ satisfies $f(x_\lambda) \to f(x_\infty)$.

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Consider the image $f(x_\infty) \in Y$.

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Consider the image $f(x_\infty) \in Y$.

Let $V \in \tau_Y$ be such that $f(x_\infty) \in V$.

By continuity, $U = f^{-1}(V)$ is open in X.

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Observe that x_∞ belongs to U.

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The net $f(x_\lambda)$ is in $Y - V$, which is a closed set.

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There is a λ_U such that $\lambda_U \preceq \lambda$ implies $x_\lambda \in U$.

It follows that $\lambda_U \preceq \lambda$ implies $f(x_\lambda) \in V$.

Since V is arbitrary, this shows that $f(x_\lambda) \to f(x_\infty)$.

Conversely, suppose f is not continuous so that $V \in \tau_Y$ has $A = f^{-1}(V) \notin \tau_X$.

There is a net $x_\lambda \in X - A$ such that $x_\lambda \to x_\infty \in A$.

The net $f(x_\lambda)$ is in $Y - V$, which is a closed set.

This net cannot converge to $f(x_\infty) \in V$.

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A topological space is said to be sequentially compact if every sequence has a convergent subsequence.

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There are compact topological spaces that are not sequentially compact.

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There are sequentially compact topological spaces that are not compact.

Suppose $(D_0, \preceq_0)$ and $(D_1, \preceq_1)$ are two directed sets.

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Suppose $(D_0, \preceq_0)$ and $(D_1, \preceq_1)$ are two directed sets.

A function $f : D_0 \to D_1$ preserves order if $x \preceq_0 y$ implies $f(x) \preceq_1 f(y)$, and is cofinal if to each $y \in D_1$ there is an $x_y \in D_0$ such that $y \preceq_1 f(x_y)$.

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A net $x : D_1 \to X$ has a subnet $x \circ f : D_0 \to X$ whenever f preserves order and is cofinal.

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A topological space X is compact if and only if each net has a convergent subnet.

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The following reasoning is flawed.

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"A sequence may be regarded as a net.

Inside a compact space there is therefore a convergent subnet.

This produces a convergent subsequence, and hence every compact space is sequentially compact."

The subtlety is that a subnet of a sequence need not be a subsequence, f need not be 1-1.

Suppose X is compact and let $A \subset X$ be closed.

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Closed subsets of compact spaces are always compact.

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This is an open cover of X.

Let $V_{\alpha_1}, \ldots, V_{\alpha_n}$ together with $X - A$ be a finite subcover.

Toss out $X - A$ and conclude that $U_{\alpha_1}, \ldots, U_{\alpha_n}$ is a finite subcover of A.

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Recall that a net $x : D \to X$ converges to $x_\infty \in X$ if to each open set $U \in \tau$ such that $x_\infty \in U$, there is a $\lambda_U \in D$ such that $\lambda_U \preceq \lambda$ implies $x_\lambda \in U$.

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This may be expressed by the phrase "the net x is eventually in U".

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A point $x_\infty \in X$ is a cluster point of the net x if to each open set $U \in \tau$ such that $x_\infty \in U$ it is true that given any $\lambda_0 \in D$ there is some $\lambda \in D$ with $\lambda_0 \preceq \lambda$ and $x_\lambda \in U$.

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A collection of sets with the property that any finite sub-collection has nonempty intersection is said to satisfy the finite intersection property. **Theorem:** The following are equivalent properties of a topological space (X, τ).

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(1) Each net has a convergent subnet.

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A collection of sets with the property that any finite sub-collection has nonempty intersection is said to satisfy the finite intersection property. **Theorem:** The following are equivalent properties of a topological space (X, τ).

(1) Each net has a convergent subnet.

(2) Each net has a cluster point.

(3) Each open cover has, a finite subcover.

(4) Each collection of closed sets that satisfies the finite intersection property has nonempty intersection.

Proof:

$(1) \Rightarrow (2)$

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There is a $\lambda_U^0 \in D_0$ such that $\lambda_U^0 \preceq \lambda^0 \Rightarrow y_{\lambda^0} \in U$.

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There is a $\lambda^0_U \in D_0$ such that $\lambda^0_U \preceq \lambda^0 \Rightarrow y_{\lambda^0} \in U$.

Given any $\lambda^1 \in D_1$, there is a $\lambda^0_{\lambda^1} \in D_0$ such that $\lambda^0_{\lambda^1} \preceq \lambda^0 \Rightarrow \lambda^1 \preceq f(\lambda^0)$.

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There is now a $\lambda^0_0 \in D_0$ such that $\lambda^0_U \preceq \lambda^0_0$ and $\lambda^0_{\lambda^1} \preceq \lambda^0_0$.

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There is now a $\lambda^0_0 \in D_0$ such that $\lambda^0_U \preceq \lambda^0_0$ and $\lambda^0_{\lambda^1} \preceq \lambda^0_0$.

Let $\lambda^1_0 = f(\lambda^0_0)$ and note that $x_{\lambda^1_0} \in U$.

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Write $(\lambda_0, U_0) \preceq_R (\lambda_1, U_1)$ if and only if $\lambda_0 \preceq \lambda_1$ and $U_0 \supset U_1$.

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Choose any $\lambda_0 \in D$ such that $x_{\lambda_0} \in U$.

If $(\lambda_0, U) \preceq_R (\lambda, U_0)$, then

$$y_{(\lambda, U_0)} = x_\lambda \in U_0 \subset U.$$

$(3) \Rightarrow (4)$

(3)⇒(4)

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Taking complements yields a contradiction.

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$$x_\infty \in \bigcap_{\lambda \in D} A_\lambda^-.$$

If x_∞ is not a cluster point of x, then there is some open set U containing x_∞ and some $\lambda \in D$ such that

$$\lambda \preceq \lambda_0 \Rightarrow x_{\lambda_0} \notin U.$$

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$$A_\lambda \cap U = \emptyset,$$

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There are minimizing sequences x_k with $f(x_k) < \alpha + \frac{1}{k}$.

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This certainly seems possible unless there is some additional assumption on f.

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The only resolution is to have $f(x_\infty) = \alpha$, which may expressed by asserting that the minimum is attained at $x_\infty \in X$.

The combination compact space and semi-continuous function is therefore an attractive way to guarantee the existence of an optimal point.

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Let $\phi : [0,1] \to [0,1)$ be given by $\phi(1) = 0$ and then use your favorite $1-1$ and onto function from $[0,1)$ to $(0,1)$.

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Since $[0,1]$ is compact, a topological property, this implies that each such function is not continuous or its inverse is not continuous.

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The unit circle $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ is compact whereas the real line $\mathbb{R}$ is not compact, so as expected the two spaces are not homeomorphic.

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It is however true that the continuous image of a compact set must be compact.

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It follows that $f(A)$ is compact.

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Suppose $f : X \to Y$ is continuous and $C \subset Y$ closed.

Let $x_\lambda \to x_\infty$ with $x_\lambda \in f^{-1}(C)$.

The continuity implies $f(x_\lambda) \to f(x_\infty)$.

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For continuous functions the inverse image of a closed set is always closed.

Suppose $f : X \to X$ is continuous.

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In Hausdorff spaces the set of fixed points of a continuous function is always closed.

Let X be a set and $\mathcal{T}$ the collection of all topologies on X.

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Consider a subset S of a given topology τ, so S is a collection of open sets with $S \subset \tau$ rather than an open set, which in this case would mean $S \in \tau$.

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The discrete topology contains S.

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The topology τ_S consists of arbitrary unions of finite intersections of members of S and the whole set if not included already.

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Some lines of reasoning may simplify as the conditions involved are only necessary to verify on a basis or maybe even a subbasis.

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Let (X, τ_0) and (X, τ_1) be two topological spaces with the same underlying set of points X.

If $\tau_0 \subset \tau_1$, then τ_0 is said to be weaker than τ_1, and τ_1 is said to be stronger than τ_0.

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The term finer is sometimes used in place of stronger, and coarser is sometimes used in place of weaker.

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A collection of functions $F = \{f_\alpha : S \to (X_\alpha, \tau_\alpha)\}$ may be used to introduce a topology on the set S, the domain of the functions.

Observe that the set of subsets, $P(S)$, would render each f_α continuous.

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Let τ_F be the intersection of all topologies that render each f_α continuous.

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The topology τ_F is known as the weak topology on S induced by F.

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Observe that the set of subsets, $P(S)$, would render each f_α continuous.

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One has $s_\lambda \to s_\infty$ if and only if $f_\alpha(s_\lambda) \to f_\alpha(s_\infty)$ for all α.

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It may be that S already has a topology.

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Let S be a normed vector space, and let F be the collection of continuous linear functionals on S.

There are now two topologies on S, the metric topology induced by the norm and the weak topology induced by F.

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Let S be a normed vector space, and let F be the collection of continuous linear functionals on S.

There are now two topologies on S, the metric topology induced by the norm and the weak topology induced by F.

The terminology is consistent here since it is possible to show that the weak topology in fact is weaker than then norm topology.

Recall that one writes

$$X_1 \times \cdots \times X_n = \{(x_1, \ldots, x_n) \mid x_i \in X_i, \}\,.$$

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If the space involved have topologies, then the weak topology associated with the projections is known as the product topology on $X_1 \times \cdots \times X_n$.

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If X_i are compact, then $X_1 \times \cdots \times X_n$ is compact.

If U_i is open in X_i, then $\pi_i^{-1}(U_i)$ is open in $X_1 \times \cdots \times X_n$.

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The collection of all such open sets using all the projections is a subbasis for the product topology.

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An open rectangle in $\mathbb{R}^2$ is produced by $\pi_1^{-1}((a,b)) \cap \pi_2^{-1}((c,d))$.

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Let $X = \{f : [0,1] \to \mathbb{R} \mid f \text{ continuous}\}$ and consider the collection of functions $\varphi_t : X \to \mathbb{R}$ given by $\varphi_t(f) = f(t)$.

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The topology τ is not the discrete topology.

Since X is not finite it is possible that τ is metrizable nonetheless.

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It turns out that (X, τ) is in fact not metrizable.

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This produces a natural example of a topology that cannot be defined by a metric.

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The φ_k is sometimes known as the projection onto the k'th coordinate.

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$$d(f,g) = \sum_{k=1}^{\infty} \frac{\min\{|f(k) - g(k)|, 1\}}{2^k}.$$

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The use of the intersection of family of topologies, where each member is somewhat mysterious, creates an object that is difficult to understand.

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For a general family $\varphi_\alpha : X \to X_\alpha$ various sets $\varphi_\alpha^{-1}(U_\alpha)$, where U_α is open in X_α, form a subbasis for the weak topology.

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Consider the case $\mathbb{R}^2$ and the vertical open strips of the form

$$\left\{(x, y) \in \mathbb{R}^2 \mid a < x < b \ \&\ y \in \mathbb{R}\right\},$$

and their horizontal counter parts.

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It is of course necessary to use infinitely many strips to create an open disk.

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Declare $U \subset X/\sim$ open if and only if $\pi^{-1}(U) \in \tau$.

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The inverse image preserves both unions and intersections so this indeed a topology known as the identification topology.

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Suppose $A \subset X$ and partition X into singletons $\{x\}$, when $x \notin A$, together with the set A itself.

The associated equivalence relation produces a space $X/\sim$ that frequently is written X/A.

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The map is in fact a set-isomorphism.

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If $[x_\infty] = [0]$, then it must be understood that the relevant open sets have the form $U = [0,\beta) \cup (\alpha, 1]$ and not only one of the two sets in the union.

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Keeping this in mind, it is seen that $\phi([x_\lambda]) \to 1 = \phi([x_\infty])$.

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Now look at ϕ^{-1}.

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If $z_\infty \neq 1$, then eventually $z_\lambda = e^{2\pi i x_\lambda}$ with $x_\lambda \in (0,1)$.

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If $z_\infty \neq 1$, then eventually $z_\lambda = e^{2\pi i x_\lambda}$ with $x_\lambda \in (0,1)$.

Moreover, $x_\lambda \to x_\infty$ where $z_\infty = e^{2\pi i x_\infty}$.

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Finally, if $z_\infty = 1$ then eventually $z_\lambda = e^{2\pi i x_\lambda}$ has $x_\lambda \in U$ no matter what the feasible choice is of $0 < \beta < \alpha < 1$.

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So $\phi^1(z_\lambda) \to \phi^{-1}(z_\infty)$ in either case, and hence ϕ^{-1} is continuous.

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Putting things together, $[0,1]/\{0,1\}$ is homeomorphic to S^1.