An Algorithm for Generalized Sylvester-observer Equation in State Estimation of Descriptor Systems

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Abstract

An algorithm for solving the generalized Sylvester-observer equation arising in state estimation of descriptor systems is proposed. The algorithm is structure preserving and rich in BLAS-3 computations and thus suitable for high performance computing. The algorithm is a generalization of the one recently proposed by Datta and Sarkissian in state-estimation of standard first-order state space systems. The algorithm can be used in state estimation of matrix second-order control systems, and an algorithm for this purpose is given.

1 Introduction

In this paper, we consider the problem of state estimation of the linear time-invariant continuous-time generalized state space system, also called *descriptor system*

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + B_0u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $E, A \in \mathbb{R}^{n\times n}$, $B_0 \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{r\times n}$ and $rank(E) \geq n - r$.

If E is nonsingular, the system (1) has the standard first-order state space form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \tilde{B}u(t) \\ y(t) &= Cx(t). \end{aligned} \tag{2}$$

Descriptor systems arise, for example, in modeling interconnected systems, constrained mechanical systems and economic processes [4].

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A control system represented by a system of matrix second-order differential equations of the form:

$$\begin{aligned} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) &= Bu(t) \\ y(t) &= C_1q(t) + C_2\dot{q}(t), \end{aligned} \tag{3}$$

arising in stabilization and control of vibrating structures such as bridges, highways, automobiles and aircrafts, can be casted into the descriptor system form (1) where $x = \begin{bmatrix} \dot{q} & q \end{bmatrix}^T$ and

If M is nonsingular:

$$\begin{aligned} E &= \begin{bmatrix} 0 & M \\ M & D \end{bmatrix}, \; A = \begin{bmatrix} M & \\ & -K \end{bmatrix} \\ B_0 &= \begin{bmatrix} 0 \\ B \end{bmatrix}, \; C = \begin{bmatrix} C_2 & C_1 \end{bmatrix}. \end{aligned} \tag{4}$$

Otherwise, if K is nonsingular:

$$\begin{aligned} E &= \begin{bmatrix} M & \\ & -K \end{bmatrix}, \; A = \begin{bmatrix} -D & -K \\ -K & 0 \end{bmatrix} \\ B_0 &= \begin{bmatrix} B \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} C_2 & C_1 \end{bmatrix}. \end{aligned} \tag{5}$$

There exist in the literature a few numerical algorithms [16, 17, 20, 21, 22] for computations of transfer functions and pole assignment of descriptor systems. The work on state estimation on such systems include [6, 12]. An algorithm for reduced order observer of such systems appears in [13].

The *Sylvester-observer equation approach* [4, 10] for state estimation of the system (2) is based on numerically solving the matrix equation

$$XA - FX = GC, \tag{6}$$

which is a variation of the well known Sylvester equation $XA - BX = C$. Equation (6) is called

Sylvester-observer equation [2, 8, 10]. In this equation, A and C are known and X, F and G are to be found. If F is a stable matrix, then it can be shown [4, 10] that $e(t) = z(t) - Xx(t) \to 0$ as $t \to \infty$, where $z(t)$ satisfies the system of differential equations:

$$\dot{z}(t) = Fz(t) + Gy(t) + XBu(t) \ , \ z(0) = 0. \quad (7)$$

Here $y(t)$ is the observed output of the system for a chosen input $u(t)$. An estimate $\hat{x}(t)$ to the state vector $x(t)$ can be computed [15] by solving

$$\begin{bmatrix} X \\ C \end{bmatrix} \hat{x}(t) = \begin{bmatrix} z(t) \\ y(t) \end{bmatrix}. \quad (8)$$

It is well known [4] that (6) can be solved for X, F and G if $\Omega(A) \bigcap \Omega(F) = \emptyset$, where $\Omega(M)$ denotes the spectrum set of a matrix M. In order to solve (8), the computed matrix X must have full rank. For this to happen, two necessary conditions must be imposed [18]: the pair (A, C) must be observable and the pair (F, G) must be controllable.

The existing numerical methods for solving the Sylvester-observer equation (6) include :

- the recursive method by Van Dooren [19]
- the block generalization of Van Dooren's algorithm by Carvalho and Datta [3]
- the SVD-based algorithm by Datta and Sarkissian [9]
- the large-scale algorithms by Datta and Saad [7] and Calvetti, Lewis and Reichel [2]
- the parallel algorithm by Bischof, Datta and Purkayastha [1].

In this paper we propose a Sylvester-observer equation approach for state estimation of the descriptor system (1). In Section 2, we prove a theorem (Theorem 2.1), analogous to the Observer Theorem [4, 9], for state estimation of the standard state space system (2), showing how the *generalized Sylvester-observer equation* can be used to estimate the states of the descriptor system (1).

In Section 3, an SVD-based block algorithm is proposed to solve this generalized Sylvester-observer equation. The proposed algorithm generalizes a recent SVD-based algorithm of Datta and Sarkissian [9]. The proposed algorithm is structure preserving and rich in BLAS-3 computations; thus, it is suitable for high performance computing [11] using the well-known package LAPACK.

2 State Estimation of Descriptor Systems

The analogue of the Sylvester-observer equation (6) for the descriptor system (1) is

$$XA - FXE = GC. \quad (9)$$

In (9) the matrices $E, A \in \mathbb{R}^{n \times n}$ and $C \in \mathbb{R}^{r \times n}$ are given and the matrices $X \in \mathbb{R}^{(n-r) \times n}, F \in \mathbb{R}^{(n-r) \times (n-r)}$ and $G \in \mathbb{R}^{(n-r) \times r}$ are to be found.

The *observer* system still satisfies (7) and an estimate $\hat{x}(t)$ is computed by solving the linear system

$$\begin{bmatrix} XE \\ C \end{bmatrix} \hat{x}(t) = \begin{bmatrix} z(t) \\ y(t) \end{bmatrix}. \quad (10)$$

In order to solve (10), the matrix XE must have full rank. Therefore, the solution matrix X of (9) is desired to have full rank $n - r$ and we must assume $rank(E) \geq n - r$. In this context, it is noted that necessary conditions for a solution X of (9) to have full rank are that the system (1) has $n - r$ observable states and that the pair (F, G) is controllable. For simplicity, we assume here that (1) is observable and (F, G) is controllable.

Also, it is noted [5] that (9) can be solved by X, F and G if $\Omega(A, E) \bigcap \Omega(F) = \emptyset$, where $\Omega(A, E)$ denotes the set of generalized eigenvalues of the pencil $P(\lambda) = A - \lambda E$.

The following theorem shows that the solution $z(t)$ of (7) asymptotically approximates a linear transformation of the state vector $x(t)$ of (1) when $X \in \mathbb{R}^{(n-r) \times n}$ is the solution of (9) and matrix F is chosen to be stable (that is, all the eigenvalues have negative real parts).

Theorem 2.1: Reduced-order Observer Theorem for Descriptor systems.

The system (7) is a state-observer of the system (1), that is, $z(t)$ is an estimate of $XEx(t)$ in the sense that the error $e(t) = z(t) - XEx(t)$ approaches zero as time t increases, for any initial condition $x(0)$, if X, F and G satisfy (9) and F is stable.

Proof: Using (1) and (9) we can write

$$\begin{aligned}\dot{e}(t) &= \dot{z}(t) - XE\dot{x}(t) = Fz(t)+\\ &+GCx(t) + XBu(t) - XAx(t) - XBu(t) =\\ &= Fz(t) - FXEx(t) = F[z(t) - XEx(t)] \ .\end{aligned}$$

Therefore $e(t) = \exp(Ft)[z(0) - XEx(0)]$, where $z(0)$ and $x(0)$ are any initial conditions. Thus $e(t) \to 0$ as $t \to \infty$, since F is stable.

3 An SVD-based Method for solving $XA - FXE = GC$

In this section, we show how the algorithm presented by Datta and Sarkissian [9] can be generalized to solve the generalized Sylvester-observer equation (9). We recall that $E, A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{r\times n}$ are such that the system (1) is observable (see [14] for the observability of (1)).

Let $\mathcal{S}$ be a self-conjugate set of complex numbers such that $\Omega(A, E) \bigcap \mathcal{S} = \emptyset$. In order to solve (9) block-wise, partition the matrix F in $q \times q$ blocks,

$$F = \begin{bmatrix} F_{11} & F_{12} & & \\ & F_{22} & F_{23} & \\ & & \cdots & \cdots \\ & & & F_{qq} \end{bmatrix} \tag{11}$$

where diagonal blocks $F_{ii}, i = 1, \ldots, q$ of F are chosen to be matrices in real Schur form [10] such that the requirement $\Omega(F) = \mathcal{S}$ is satisfied. Partitioning matrices X and G accordingly with the partition of F,

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \cdots \\ X_q \end{bmatrix} \ , \ G = \begin{bmatrix} 0 \\ \cdots \\ 0 \\ G_q \end{bmatrix}. \tag{12}$$

Using (11) and (12) in (9) and equating the corresponding blocks on each side, we obtain

$$X_iA - F_{ii}X_iE = F_{i,i+1}X_{i+1}E \ , \ i = 1, \ldots, q-1 \tag{13}$$

$$X_qA - F_{qq}X_qE = G_qC. \tag{14}$$

Let $\tilde{F}_{qq}$ be a matrix with a prescribed spectrum and let Y_q satisfy the Sylvester equation

$$Y_qA - \tilde{F}_{qq}Y_qE = C \tag{15}$$

Taking now the SVD of Y_q^T: $Y_q^T = U_q\Sigma_qV_q^T$ gives

$$V_q\Sigma_qU_q^TA - \tilde{F}_{qq}V_q\Sigma_qU_q^TE = C.$$

If Y_q has full rank, we have

$$U_q^TA - (\Sigma_q^{-1}V_q^T\tilde{F}_{qq}V_q\Sigma_q)U_q^TE = \Sigma_q^{-1}V_q^TC \tag{16}$$

meaning that we have a solution of (14) with

$$X_q = U_q^T \ , \ F_{qq} = \Sigma_q^{-1}V_q^T\tilde{F}_{qq}V_q\Sigma_q \ , \ G_q = \Sigma_q^{-1}V_q^T. \tag{17}$$

This strategy can be repeated for each $i = q-1, \ldots, 2, 1$ to obtain a solution of (13) as follows: Choosing $\tilde{F}_{ii}$ with a prescribed spectrum, we solve

$$Y_iA - \tilde{F}_{ii}Y_iE = X_{i+1}E \tag{18}$$

for Y_i and compute the SVD of Y_i^T : $Y_i^T = U_i\Sigma_iV_i^T$. If Y_i has full rank, compute

$$X_i = U_i^T \ , \ F_{ii} = \Sigma_i^{-1}V_i^T\tilde{F}_{ii}V_i\Sigma_i \ , \ F_{i,i+1} = \Sigma_i^{-1}V_i^T. \tag{19}$$

Our discussion leads to the following algorithm:

Algorithm 3.1 : Block Algorithm for the generalized Sylvester-observer equation

Input: Matrices $E \in \mathbb{R}^{n\times n}, A \in \mathbb{R}^{n\times n}, C \in \mathbb{R}^{r\times n}$ and self conjugate stable vector $\mathcal{S} \in \mathbb{C}^{n-r}$.

Output: matrices $X \in \mathbb{R}^{(n-r)\times n}$, $F \in \mathbb{R}^{(n-r)\times(n-r)}$ and $G \in \mathbb{R}^{(n-r)\times r}$ such that $XA - FXE = GC$ and $\Omega(F) = \mathcal{S}$.

Assumptions: The system (1) is observable, C has full rank and $\Omega(A, E) \bigcap \mathcal{S} = \emptyset$.

Step 1: Distribute the elements of $\mathcal{S}$ in self-conjugate groups $S_i, i = 1, \ldots, q$.

Step 2: For $i = q, q-1, \ldots, 1$ do Steps 3 to 9

Step 3: If $i = q$ set $J = C$; else set $J = X_{i+1}E$.

Step 4: Choose $\tilde{F}_{ii}$ to be a matrix in real Schur form such that $\Omega(\tilde{F}_{ii}) = S_i$.

Step 5: Solve, for Y_i, the generalized Sylvester equation $Y_iA - \tilde{F}_{ii}Y_iE = J$ using available standard software.

Step 6: From the economy size SVD of Y_i^T, compute matrices U_i and V_i and diagonal matrix Σ_i such that $Y_i^T = U_i\Sigma_iV_i^T$.

Step 7: If Σ_i is nonsingular, compute

$$F_{ii} = \Sigma_i^{-1}V_i^T\tilde{F}_{ii}V_i\Sigma_i$$

taking advantage of the structure of $\tilde{F}_{ii}$. Otherwise, find another set S_i and go back to Step 5.

Step 8: If $i = q$, compute $G_q = \Sigma_q^{-1} V_q^T$; else, compute $F_{i,i+1} = \Sigma_i^{-1} V_i^T$.

Step 9: Set $X_i = U_i^T$.

Step 10: Form matrices X, F and G, from their blocks computed above.

Remarks;
1. As in [9], the above algorithm produces a matrix X such that every block-row $X_i, i = 1, \ldots, q$ is a matrix whose rows form an orthonormal set.

2. The algorithm does not require reduction of the matrices E, A and C. This is specially attractive when E and A are large and sparse, as long as we are able to exploit this structure in Step 5.

3. If the matrices E and A are dense, then the pair (E, A) can be reduced to (T, H), where H is a block upper Hessenberg matrix and T is upper triangular. The advantage of this block Hessenberg-triangular form can again be exploited in Step 5.

4. The algorithm is rich in BLAS-3 computations and thus is suitable for high-performance computing [11] using LAPACK.

4 Reduced-order State Estimation for Matrix Second-order Systems

The results of the previous section can be conveniently used to estimate the states of the second-order observable system (3). To do this, we recall the first-order representation (1) obtained from the use of either (4) or (5). Suppose that the matrices $F \in \mathbb{R}^{(2n-r)\times(2n-r)}$, $X \in \mathbb{R}^{(2n-r)\times 2n}$, $G \in \mathbb{R}^{(2n-r)\times r}$ have been computed such that F is a stable matrix and (9) holds. See [14] for the observability of (3).

Then, approximations $\hat{q}(t)$ and $\hat{v}(t)$ of $q(t)$ and $\dot{q}(t)$ can be found by solving

$$\begin{bmatrix} XE \\ C \end{bmatrix} \begin{bmatrix} \hat{v}(t) \\ \hat{q}(t) \end{bmatrix} = \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} \qquad (20)$$

where $z(t) \in \mathbb{R}^{2n-r}$ is the solution of the initial value problem (7).

Algorithm 4.1: State Estimation of Matrix Second-order Systems.

Input: System matrices M, D, K, B, C_1 and C_2 of the system (3).

Output: Estimates $\hat{q}(t)$ and $\hat{v}(t)$ of $q(t)$ and $\dot{q}(t)$ respectively.

Assumption: The system (3) is observable, C_1 and C_2 are matrices with full rank and least one of the matrices M and K is nonsingular.

Step 1: If M is nonsingular, compute matrices E, A, B_0 and C using (4), otherwise, use (5).

Step 2: Solve the generalized Sylvester-observer equation (9) for a stable matrix F.

Step 3: Compute the vector $z(t)$ from (7).

Step 4: Compute $\hat{q}(t)$ and $\hat{v}(t)$ from (20).

5 Numerical Examples

Example 1: Consider the second-order system (3) with

$$M = diag\begin{pmatrix} 2.9550 & 3.5220 & 0.5710 \end{pmatrix}$$

$$D = \begin{bmatrix} 193.4974 & -1.1174 & 0 \\ -1.1174 & 40.1125 & -2.6801 \\ 0 & -2.6801 & 50.1971 \end{bmatrix}$$

$$K = \begin{bmatrix} 382.3000 & -382.3000 & 0 \\ -382.3000 & 432.8500 & -50.5500 \\ 0 & 50.5500 & -50.5500 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1.9200 & 19.8400 & 32.7200 \\ 53.3700 & 56.0300 & 84.1100 \end{bmatrix}.$$

Solving the generalized Sylvester-observer equation (9) using Algorithm 3.1, we obtain

$$X = \begin{bmatrix} .0003 & -.9960 & .0664 & .0353 & .0206 & .0427 \\ -.8329 & -.0187 & -.3331 & .1653 & -.3858 & .1370 \\ -.3052 & .4943 & -.8086 & .0299 & -.0460 & .0747 \\ -.9060 & .0215 & .3763 & .1083 & -.0931 & .1289 \end{bmatrix}$$

$$F = \begin{bmatrix} -10.0000 & 1.6329 & 4.3713 & -0.4391 \\ -61.2406 & -10.0000 & 2.6890 & 26.7700 \\ 0 & 0 & -11.3836 & 0.3206 \\ 0 & 0 & 37.1845 & -18.6164 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.1568 & 0.3912 \\ -4.2136 & 1.6884 \end{bmatrix}$$

where F is a stable matrix with chosen eigenvalues $\mathcal{S} = \{\ -10 \quad -10-10i \quad -10+10i \quad -20\ \}$.

Figure 1 shows a comparison between the third components of the actual state $\dot{q}(t)$, computed by solving (1) numerically, and the estimated derivative $\hat{v}(t)$, computed using (20). The observer vector $z(t)$ was computed using **Matlab**'s **ode23** package. It was chosen $u(t) = \delta(t) \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and $x(0) = 0$, where $\delta(t)$ is the Dirac Delta pulse.

The plot shows that the third component of the estimated derivative estimates the third component of the actual state derivative $\dot{q}(t)$ as time t increases. Similar plots can be drawn for the other components of $\hat{v}(t)$ and $\hat{q}(t)$.

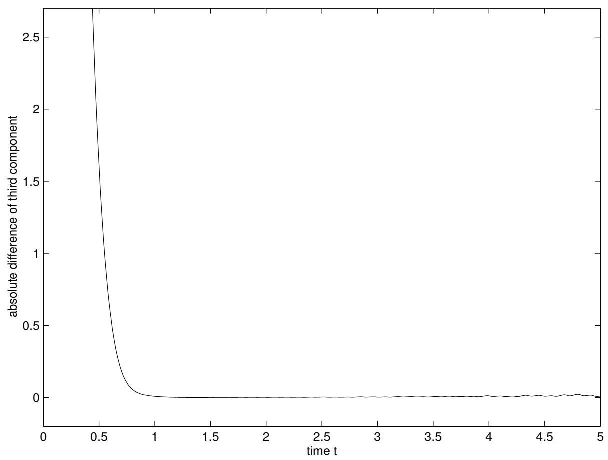

Figure 1: Plot of the absolute difference between the third components of $\hat{v}(t)$ and $\dot{q}(t)$.

Example 2: In order to estimate the state of a singular descriptor system, consider (1) with

$$E = diag \begin{pmatrix} 8 & 6 & 4 & 2 & 1 & 0 \end{pmatrix}$$

$$C = \begin{bmatrix} 0.8462 & 0.2026 & 0.8381 & 0.6813 & 0.8318 & 0.7095 \\ 0.5252 & 0.6721 & 0.0196 & 0.3795 & 0.5028 & 0.4289 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.45 & -0.04 & 0.42 & -0.09 & -0.36 & -0.48 \\ -0.27 & -0.48 & 0.24 & 0.39 & -0.30 & 0.25 \\ 0.11 & 0.32 & -0.32 & -0.44 & -0.30 & -0.05 \\ -0.01 & -0.06 & -0.09 & -0.15 & 0.10 & 0.43 \\ 0.39 & 0.12 & 0.44 & 0.31 & -0.23 & -0.03 \\ 0.25 & 0.30 & 0.42 & -0.49 & -0.30 & -0.08 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where matrix A is nonsingular but $rank(E) = 5$. Solving (9), using Algorithm 3.1, we obtain

$$X = \begin{bmatrix} .0345 & .0610 & .0811 & -.1795 & -.1976 & -.9577 \\ .0953 & .6812 & -.5731 & -.3357 & .2928 & .0008 \\ .0071 & .0100 & .0200 & -.0353 & -.0369 & -.9984 \\ .1220 & .2251 & .2725 & -.6288 & -.6794 & .0559 \end{bmatrix}$$

$$F = \begin{bmatrix} -4.0000 & 0.0260 & 1.1930 & 0.7271 \\ -616.2763 & -4.0000 & -112.0226 & 183.8075 \\ 0 & 0 & -6.6134 & 0.0387 \\ 0 & 0 & 93.6387 & -5.3866 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0625 & 0.0456 \\ -2.2408 & 3.0764 \end{bmatrix}$$

where the matrix F is stable with spectrum $\mathcal{S} = \{\ -4 \quad -4-4i \quad -4+4i \quad -8\ \}$.

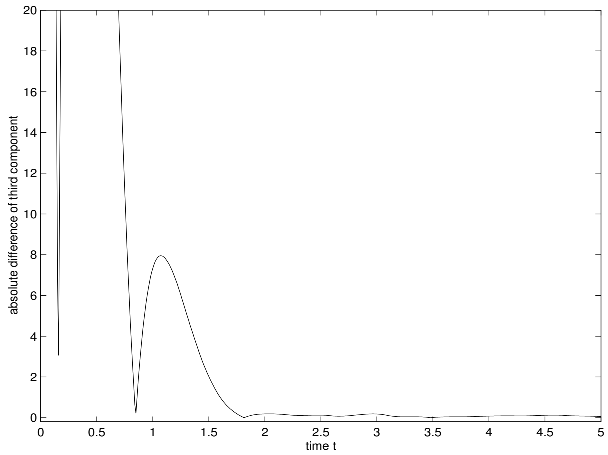

Figure 2: Plot of the absolute difference between the third components of $\hat{x}(t)$ and $x(t)$.

In order to avoid numerical difficulties when solving the singular system (1) for comparison purpose, we aim to estimate its constant solution $x(t) \equiv -A^{-1}Bu(t)$ corresponding to the constant input $u(t) \equiv \begin{bmatrix} 1 & 1 \end{bmatrix}^T$.

Figure 2 shows a comparison between the third components of the actual state $x(t)$ and the estimated state $\hat{x}(t)$, computed using (10). Again, **Matlab**'s **ode23** package and with initial condition $x(0) = 0$ where used. The plot shows that the third component of the estimated state $\hat{x}(t)$ approaches the third component of the actual state $x(t)$ as time t increases. Similar plots can be drawn for the other components of $\hat{x}(t)$ and $x(t)$.

6 Conclusions

Descriptor systems arise in many practical applications. Although numerical methods for control problems in standard state-space systems are well developed, only a few of them have so far been extended for descriptor systems . There is certainly a need for more numerically viable algorithms for descriptor control problems.

This paper proposes a numerical algorithm in state-estimation of time-invariant continuous-time descriptor systems. The algorithm, which is developed by generalizing the Sylvester equation approach for the problem in standard state-space systems, is rich in BLAS-3 computations and thus is suitable for high-performance computing by taking advantage of the software package LAPACK.

As a by-product, an algorithm for state-estimation of continuous-time matrix second-order system is obtained. Investigation into certain theoretical, computational and implementational issues, including numerical robustness of the algorithm and implementation on high-performance computers using large-scale real-life data is in order.

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