A COMPUTATIONAL METHOD FOR FEEDBACK CONTROL IN DISTRIBUTED PARAMETER SYSTEMS

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Abstract. A novel computational method for feedback control in distributed parameter systems, modeling a wide range of vibrating structures, is proposed.
Unlike in standard engineering practice, the method allows the computations to be carried out in the distributed parameter setting itself, without finite-element discretization to a matrix second-order model. Furthermore, the method can be numerically implemented using only finite-dimensional control and numerically viable computational techniques. Thus the method is practical for control, and stabilization of large vibrating structures.

Key Words Distributed parameter systems, computation, feedback control, partial eigenvalue assignment.

1. INTRODUCTION

The natural model for the vibrating systems, arising in a wide range of applications, especially in the design and analysis of vibrating structures, such as bridges, highways, buildings, airplanes, etc., is a homogeneous distributed-parameter system (DPS) of the form

$$\mathbf{M}(x)\frac{\partial^2 \nu(t,x)}{\partial t^2} + \mathbf{C}(x)\frac{\partial \nu(t,x)}{\partial t} + \mathbf{K}(x)\nu(t,x) = 0, \tag{1.1}$$

where $\mathbf{M}, \mathbf{C} = \mathbf{D} + \mathbf{G}$ and $\mathbf{K}$ are differential operators in the x-domain (spatial domain) of the displacement function $\nu(t,x)$, where for all the t the $\nu(t,x)$ belong to some Hilbert space $\mathbb{H}$, that accounts for the boundary conditions of (1.1). The operators $\mathbf{M}, \mathbf{K}, \mathbf{D}$ and $\mathbf{G}$ are, respectively, called *mass, stiffness, damping and gyroscopic* operators. In many practical applications, $\mathbf{M}$ is self-adjoint and positive definite, $\mathbf{D}$ is self-adjoint and $\mathbf{G}$ is skew-symmetric.

Though it is desirable to solve a vibration problem in its own natural distributed parameter setting, very often in practice, due to lack of effective numerical methods to handle a distributed parameter problem directly, the system (1.1) is discretized to a finite-dimensional matrix second-order system of the form:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = 0, \tag{1.2}$$

where $M, C = D + G, K \in \mathbb{R}^{n\times n}$ and $\dot{x}(t)$, respectively, denote the first and second derivatives of the time dependent vector $x(t)$.

A vibration control problem is solved and a control law is implemented on the discretized system (1.2) and then applied to a real-life vibrating structure modeled by the DPS (1.1).

A solution obtained this way naturally suffers from discretization error and after all, such a solution is just a finite-dimensional approximation of an infinite dimensional problem. Unfortunately, computational methods for solving vibration control problems using even this second-best alternative are not very well established. There are two standard approaches: solutions via a first-order realization and Independent Modal Space Control (IMSC) approach. Both of these approaches have severe engineering and computational disadvantages (See [3] and [14]).

In the last few years, the author and his collaborators have developed a novel approach, called "Direct and Partial Modal" approach for solving important feedback control problems in matrix second-order and distributed parameter systems [see (1, 4-13)]. It is direct, because the problem is solved directly in its given mathematical formulation; that is, in case the model is a discretized second-order model, no transformation to a first-order realization is invoked, and if the model is a DPS, then no discretization to a second-order system is necessary. It is partial-modal, because only a part of the spectrum and the corresponding eigenvectors (eigenfunctions) of the associated quadratic eigenvalue problem are necessary in implementing the scheme.

The direct and partial-modal nature of these feedback algorithms make them suitable for practical use in stabilization and control of large vibrating structures, such as Large Space Structures (LSS), power systems, computer networks, aircrafts and others.

In this paper, we illustrate this new approach (in algorithmic form) in case the feedback problem is the partial eigenvalue assignment in DPS. The results on numerical experiment with a real-life example are given.

2. PARTIAL EIGENVALUE ASSIGNMENT IN DPS

The Partial Eigenvalue Assignment (PEVA) problem in the DPS (1.1) is the problem of re-assigning a small part of the spectrum, responsible for undesirable dangerous responses, such as resonance or instability, of the associated open-loop operator pencil

$$P(\lambda) = \lambda^2 M + \lambda C + K, \tag{2.1}$$

by a suitable feedback, in such a way that the remaining large part is not affected.

The problem is certainly practical in the sense that in most practical situations dealing with large problems, only a small part of the spectrum is troublesome, and thus, it makes sense to change that part only by feedback without solving a large full-order eigenvalue assignment problem. Furthermore, solving a large eigenvalue assignment is unpractical and the existing numerical methods are not suitable for large and sparse problems (see [3]).

Mathematically, the PEVA in DPS is defined as follows:

Given the DPS (1.1) with M nonsingular, and the self-conjugate set $\{\mu_1, \cdots, \mu_p\}$ of scalars and suitable control functions $b_1, \cdots, b_m$. Find: real feedback functions $\mathbf{f}_{11}, \cdots, \mathbf{f}_{1m}$ and $\mathbf{f}_{21}, \cdots, \mathbf{f}_{2m}$ such that the spectrum of the closed-loop pencil

$$\begin{aligned}\mathbf{P}_c(\lambda)\phi = \lambda^2\mathbf{M}\phi + \lambda\left(\mathbf{C}\phi - \textstyle\sum_{k=1}^m(\mathbf{f}_{1k},\phi)\mathbf{b}_k\right) \\ + \left(\mathbf{K}\phi - \textstyle\sum_{k=1}^m(\mathbf{f}_{2k},\phi)\mathbf{b}_k\right)\end{aligned} \tag{2.2}$$

is the set $S = \{\mu_1, \cdots, \mu_p; \lambda_{p+1}, \lambda_{p+2}, \cdots\}$ where $\{\lambda_1, \lambda_2, \cdots, \lambda_p;\ \ \lambda_{p+1}, \lambda_{p+2}\cdots\}$ is the infinite spectrum of the open-loop pencil (2.1).

We now present a solution of the above problem in algorithmic form. The proof of the algorithm is skipped here for space limitations. It will appear elsewhere. The proof is based on an orthogonality relation between the eigenfunctions of the pencil (2.1), proved in [10].

Algorithm 2.1 (Parametric Solution to the Partial Eigenvalue Assignment Problem in Distributed Parameter System)

Inputs:

(a) The differential operators $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ of the open-loop pencil (2.1).

(b) The m control functions $\mathbf{b}_1, \ldots, \mathbf{b}_m$.

(c) The set of scalars $\{\mu_1, \ldots, \mu_p\}$, closed under complex conjugation.

(d) The self-conjugate subset $\{\lambda_1, \ldots, \lambda_p\}$ of the open-loop spectrum $\{\lambda_1, \lambda_2, \ldots\}$ and the associated eigenfunction set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$.

Outputs:
The feedback functions $\mathbf{f}_1, \ldots, \mathbf{f}_m$ and $\mathbf{f}_{21}, \ldots, \mathbf{f}_{2m}$ such that the spectrum of the closed-loop operator pencil (2.2) is the set $\{\mu_1, \ldots, \mu_p;\ \lambda_{p+1},\ \lambda_{p+2}, \ldots\}$.

Assumptions:

(a) The control functions $\mathbf{b}_1, \ldots, \mathbf{b}_m$ are linearly independent.

(b) The open-loop quadratic operator pencil $\mathbf{P}(\lambda) = \lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K}$ with control functions $\mathbf{b}_1, \ldots, \mathbf{b}_m$ is partially controllable with respect to the eigenvalues $\lambda_1, \ldots, \lambda_p$.

(c) The sets $\{\lambda_1, \ldots, \lambda_p\}$, $\{\lambda_{p+1}, \lambda_{p+1}, \ldots\}$, and $\{\mu_1, \ldots, \mu_p\}$ are disjoint.

Step 1. Form $\Lambda_1 = \mathrm{diag}(\lambda_1, \ldots, \lambda_p)$, $\mathbf{V}_1 = (\mathbf{v}_1, \ldots, \mathbf{v}_p)$, and $\Lambda_{c1} = \mathrm{diag}(\mu_1, \ldots, \mu_p)$.

Step 2. Choose arbitrary $m \times 1$ vectors $\gamma_1, \ldots, \gamma_p$ in such a way that $\overline{\mu_j} = \mu_k$ implies $\overline{\gamma_j} = \gamma_k$ and form $\Gamma = (\gamma_1, \ldots, \gamma_p)$.

Step 3. Solve the following Sylvester equation for Z_1:

$$\Lambda_1 Z_1 - Z_1\Lambda_{c1} = \begin{pmatrix} (\mathbf{v}_1, \mathbf{b}_1) & \cdots & (\mathbf{v}_1, \mathbf{b}_m) \\ \vdots & \ddots & \vdots \\ (\mathbf{v}_p, \mathbf{b}_1) & \cdots & (\mathbf{v}_p, \mathbf{b}_m) \end{pmatrix}\Gamma\,.$$

If Z_1 is ill-conditioned, then return to Step 2 and select different $\gamma_1, \ldots, \gamma_p$.

Step 4. Solve $\Phi Z_1 = \Gamma$ for $\Phi = (\Phi_{ij})$.

Step 5. If none of the $\lambda_1, \ldots, \lambda_p$ is zero, form for all $k = 1, \ldots, m$

$$\begin{aligned}\mathbf{f}_{1k} &= \sum_{j=1}^p \overline{\phi}_{kj}\mathbf{M}^*\mathbf{v}_j, \text{ and} \\ \mathbf{f}_{2k} &= -\sum_{j=1}^p(\overline{\phi}_{kj}/\overline{\lambda}_j)\mathbf{K}^*\mathbf{v}_j\,,\end{aligned}$$

otherwise form for all $k = 1, \ldots, m$,

$$\mathbf{f}_{1k} = \sum_{j=1}^{p} \overline{\phi}_{kj} \mathbf{M}^* \mathbf{v}_j, \quad \text{and}$$

$$\mathbf{f}_{2k} = \sum_{j=1}^{p} \overline{\phi}_{kj} (\overline{\lambda}_j \mathbf{M}^* \mathbf{v}_j + \mathbf{C}^* \mathbf{v}_j).$$

Some Distinguished Features of Algorithm 2.1

- Only a small finite part of the infinite spectrum (and the associated eigenfunctions) is needed to numerically implement the algorithm; thus, making the algorithm viable for real-life practical applications.
- Mathematically, it can be shown [15] that that the algorithm produces a no spill-over. That is, the infinite number of eigenvalues of the open-loop operator pencil that are not reassigned, remain invariant under feedback.
- An infinite-dimensional control problem is solved using finite-dimensional control and numerically viable finite computational techniques (note that for real-life applications, control and computational techniques should be finite).

Specialization to the Discretized Model

If the mathematical model is the finite-element generated discretized second-order system (1.2), then the PEVA problem is defined as follows: Given the matrices $M, C,$ and K of the model (1.2), the control matrix B of order $n \times m$ and the self-conjugate set $\{\mu_1, \cdots, \mu_p\}$, find the feedback matrices F_1 and F_2 such that the closed-loop pencil $P_c(\lambda) = \lambda^2 M + \lambda(C - BF_1) + (K - BF_2)$ has the spectrum $\mu_1, \cdots, \mu_p, \ \lambda_{p+1}, \cdots, \lambda_{2n}\}$. In this case, Algorithm 2.1 reduces to the following:

Algorithm 2.2 (Parametric Solution to the Partial Eigenvalue Assignment Problem in Discretized Matrix Second-order Systems).

Inputs:

(a) The $n \times n$ matrices M, C, and K.

(b) The $n \times m$ control matrix B.

(c) The set $\{\mu_1, \ldots, \mu_p\}$, closed under complex conjugation.

(d) The self-conjugate subset $\{\lambda_1, \ldots, \lambda_p\}$ of the open-loop spectrum $\{\lambda_1, \ldots, \lambda_{2n}\}$ and the associated right eigenvector set $\{y_1, \ldots, y_p\}$.

Outputs:

The feedback matrices F_1 and F_2 such that the spectrum of the closed-loop pencil $P_c(\lambda) = \lambda^2 M + \lambda(C - BF_1) + (K - BF_2)$ is $\{\mu_1, \ldots, \mu_p; \lambda_{p+1}, \ldots, \lambda_{2n}\}$.

Assumptions:

(a) M is nonsingular and B has full rank.

(b) The quadratic open-loop pencil $P(\lambda) = \lambda^2 M + \lambda C + K$ with control matrix B is partially controllable with respect to the eigenvalues $\lambda_1, \ldots, \lambda_p$.

(c) The sets $\{\lambda_1, \ldots, \lambda_p\}$, $\{\lambda_{p+1}, \ldots, \lambda_{2n}\}$, and $\{\mu_1, \ldots, \mu_p\}$ are disjoint.

Step 1. Form $\Lambda_1 = \text{diag}(\lambda_1, \ldots, \lambda_p)$, $Y_1 = (y_1, \ldots, y_p)$, and $\Lambda_{c1} = \text{diag}(\mu_1, \ldots, \mu_p)$.

Step 2. Choose arbitrary $m \times 1$ vectors $\gamma_1, \ldots, \gamma_p$ in such a way that $\overline{\mu_j} = \mu_k$ implies $\overline{\gamma_j} = \gamma_k$ and form $\Gamma = (\gamma_1, \ldots, \gamma_p)$.

Step 3. Find the unique solution Z_1 of the Sylvester equation

$$\Lambda_1 Z_1 - Z_1 \Lambda_{c1} = Y_1^H B \Gamma .$$

If Z_1 is ill-conditioned, then return to Step 2 and select different $\gamma_1, \ldots, \gamma_p$.

Step 4. Solve $\Phi Z_1 = \Gamma$ for Φ.

Step 5. If none of the $\lambda_1, \ldots, \lambda_p$ is zero, form

$$F_1 = \Phi Y_1^H M \text{ and } F_2 = -\Phi \Lambda_1^{-1} Y_1^H K ,$$

otherwise form

$$F_1 = \Phi Y_1^H M \text{ and } F_2 = \Phi(\Lambda_1 Y_1^H M + Y_1^H C) .$$

Some Distinguished Features of Algorithm 2.2

1. The algorithm computes the solution of a large partial eigenvalue assignment problem by solving a small linear algebraic system and by using only the few eigenvalues of the associated large quadratic pencil that need to be reassigned and the corresponding right eigenvectors. Thus, the algorithm is readily applicable to control dangerous vibration in a structure, where only a small part of the spectrum needs to be reassigned and the rest is to remain unchanged. Furthermore, one can take complete advantage of the sparsity, symmetry, definiteness, etc., of the matrices M, C, and K in computing F_1 and F_2.

2. In case none of the eigenvalues to be reassigned is zero, computation of feedback matrices F_1 and F_2 do not involve the damping matrix C; which is remarkable, because it is hard to estimate damping in practice.

3. The algorithm, like Algorithm 2.1, is a parametric algorithm. It offers an opportunity to choose the parametric vectors $\gamma_1, \cdots, \gamma_p$ in such a way that the recurring feedback matrices F_1 and F_2 are as well-conditioned as possible or have minimal norms, such feedback matrices might yield to be a numerically robust closed-loop system. This is left as a future research topic.

3. A NUMERICAL EXAMPLE

We consider a gyroscopic distributed parameter system modelling the small oscillations of a uniform string traveling with constant velocity $\gamma < c$ over two fixed supports at $x = 0$ and $x = L$. With $L = 1$, $\gamma = 1/2$, and $c = 1$, the operators $\mathbf{M}$, $\mathbf{G}$, and $\mathbf{K}$, are given by

$$\mathbf{M}v = v\,,\ \mathbf{G}v = \frac{\partial v}{\partial x}\,,\ \mathbf{K}v = \frac{3}{4}\frac{\partial^2 v}{\partial x^2}\,,$$

where $v(0) = v(1) = 0$, with the scalar product $(v, w) = \int_0^1 \overline{v(x)}w(x)\,dx$. It can be easily shown that the operators $\mathbf{M}$, $\mathbf{G}$, and $\mathbf{K}$ have the following properties:

$$\mathbf{M}^* = \mathbf{M}\,,\ \mathbf{G}^* = -\mathbf{G} \text{ and } \mathbf{K}^* = \mathbf{K}\,.$$

The eigenvalues of $\mathbf{P}(\lambda)$ are: $\lambda_k = \frac{3}{4}\pi k i\,,\ k = \pm1, \pm2, \ldots$ and their corresponding left eigenfunctions are: $\mathbf{v}_{ck}(x) = e^{\frac{3}{2}\pi kix} - e^{-\frac{1}{2}\pi kix}$ where $0 \le x \le 1$. Algorithm 2.1 is used to assign λ_1 to $\mu_1 = -1 + i$ and $\overline{\lambda}_1$ to $\overline{\mu}_1$, choosing the two control functions $\mathbf{b}_1(x) = 1$ and $\mathbf{b}_2(x) = \sin(\pi x)$, where $0 \le x \le 1$. The step-wise results of our implementations are given in the following:

Step 1. $\mathbf{\Lambda}_1 = \operatorname{diag}\left(\frac{3}{4}\pi i, -\frac{3}{4}\pi i\right)$, $\mathbf{V}_1 = \left(e^{\frac{3}{2}\pi ix} - e^{-\frac{1}{2}\pi ix}, e^{-\frac{3}{2}\pi ix} - e^{\frac{1}{2}\pi ix}\right)$ and

$$\mathbf{\Lambda}_{c1} = \operatorname{diag}(-1 + i, -1 - i)\,.$$

Step 2. Let's choose

$$\Gamma = \begin{pmatrix} -.20575 + .8342i & -.20575 - .8342i \\ .22626 + .45888i & .22626 + .45888i \end{pmatrix}.$$

Step 3. Solving the Sylvester equation for Z_1,

$$\Lambda_1 Z_1 - Z_1 \Lambda_{c1} = \begin{pmatrix} (\mathbf{v}_1, \mathbf{b}_1) & (\mathbf{v}_1, \mathbf{b}_2) \\ (\mathbf{v}_2, \mathbf{b}_1) & (\mathbf{v}_2, \mathbf{b}_2) \end{pmatrix} \Gamma\,,$$

we obtain

$$Z_1 = \begin{pmatrix} .18844 + .36623i & -.14535 - .84357i \\ -.14535 + .84357i & .18844 - .36623i \end{pmatrix}.$$

Step 4. Solving $\Phi Z_1 = \Gamma$ for Φ_1, we obtain

$$\Phi = \begin{pmatrix} .82911 - .50588i & .82911 + .50588i \\ .25486 + .45099i & .25486 - .45099i \end{pmatrix}.$$

Step 5. The velocity feedback functions $\mathbf{f}_{11}$ and $\mathbf{f}_{12}$, plotted in Fig. 1, are given by

$$\begin{aligned}
\mathbf{f}_{11} &= -4.0471\cos^2\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi x}{2}\right) \\
&\quad -6.6329\cos\left(\frac{\pi x}{2}\right)\sin^2\left(\frac{\pi x}{2}\right) \\
\mathbf{f}_{12} &= 3.60796\cos^6\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi x}{2}\right) \\
&\quad -2.03892\cos^5\left(\frac{\pi x}{2}\right)\sin^2\left(\frac{\pi x}{2}\right) \\
&\quad +7.21593\cos^4\left(\frac{\pi x}{2}\right)\sin^3\left(\frac{\pi x}{2}\right) \\
&\quad -4.07785\cos^3\left(\frac{\pi x}{2}\right)\sin^4\left(\frac{\pi x}{2}\right) \\
&\quad +3.60796\cos^2\left(\frac{\pi x}{2}\right)\sin^5\left(\frac{\pi x}{2}\right) \\
&\quad -2.03892\cos\left(\frac{\pi x}{2}\right)\sin^6\left(\frac{\pi x}{2}\right)
\end{aligned}$$

and the position feedback functions $\mathbf{f}_{21}$ and $\mathbf{f}_{22}$, plotted in Fig. 2, are given by

$$\begin{aligned}
\mathbf{f}_{21} &= 6.3571\cos^3\left(\frac{\pi x}{2}\right) \\
&\quad +36.466\cos^2\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi x}{2}\right) \\
&\quad -22.25\cos\left(\frac{\pi x}{2}\right)\sin^2\left(\frac{\pi x}{2}\right) \\
&\quad -10.418\sin^3\left(\frac{\pi x}{2}\right) \\
\mathbf{f}_{22} &= -5.6673\cos^3\left(\frac{\pi x}{2}\right) \\
&\quad +11.209\cos^2\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi x}{2}\right) \\
&\quad +19.835\cos\left(\frac{\pi x}{2}\right)\sin^2\left(\frac{\pi x}{2}\right) \\
&\quad -3.2027\sin^3\left(\frac{\pi x}{2}\right)\,.
\end{aligned}$$

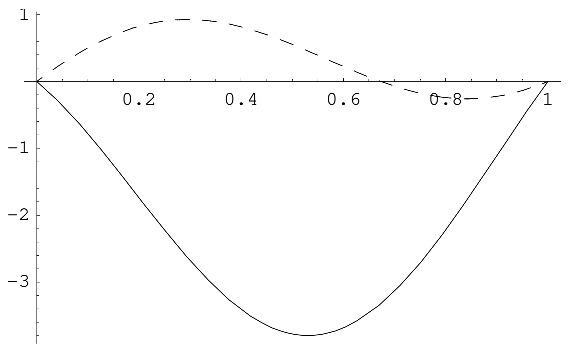

Figure 1. The Velocity Feedback Functions $\mathbf{f}_{11}$ and $\mathbf{f}_{12}$.

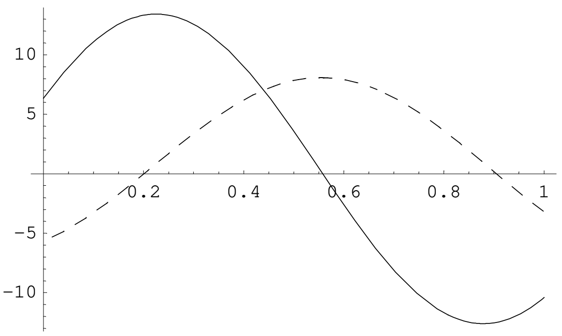

Figure 2. The Position Feedback Functions $\mathbf{f}_{21}$ and $\mathbf{f}_{22}$.

The closed-loop operator pencil $\mathbf{P}_c(\lambda)(\lambda)$ has the eigenvalues μ_1 and $\overline{\mu}_1$ with eigenfunctions given by

$$\begin{aligned}\mathbf{w}_{c1} &= (0.4171 + 0.10287i)\\ &\quad -(0.23671 + 0.20962i)e^{-2(1+i)x}\\ &\quad -(0.19416 - 0.07776i)e^{\frac{2}{3}(1+i)x}\\ &\quad -(0.0088786 - 0.037267i)e^{-\pi i x}\\ &\quad +(0.022656 - 0.0082765i)e^{\pi i x}\end{aligned}$$

and $\overline{\mathbf{w}_{c1}(x)}$, respectively. Furthermore, the eigenvalues λ_k, $k = \pm 2, \pm 3, \ldots$, of the open-loop pencil $\mathbf{P}(\lambda) = \lambda^2\mathbf{M} + \lambda\mathbf{G} + \mathbf{K}$ remain unchanged.

4. CONCLUSION.

A novel approach for feedback computations for vibrating structures, modeled by distributed parameter systems (DPS) and the associated discretized systems of matrix second-order differential equations, has been recently proposed by the author and his collaborators. The distinguished features of this approach are that it works directly with the given mathematical model, uses finite-dimensional control, computational requirements are minimal, and computational techniques are numerically sound. The feedback algorithms thus, are practical, and can be used to stabilize and control dangerous vibrations such as resonance in large vibrating structures including LSS, power systems, aircrafts, computer networks, etc.

The approach is illustrated in this paper with a computational algorithm for an important feedback problem; namely, the partial pole assignment problem in DPS.

It is believed that this new approach is poised to set a new direction of research in vibration control. Much still remains to be done in this area.

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