

The Banzhaf Power Index

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There are seven winning groups.

The Seven Winning Groups

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1.	A	B
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1.	A	B
2.	A	C

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1.	A	B	
2.	A	C	
3.	A	B	C

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1.	A	B	
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3.	A	B	C
4.	A	B	D

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2.	A	C	
3.	A	B	C
4.	A	B	D
5.	A	C	D

The Seven Winning Groups

1.	A	B		
2.	A	C		
3.	A	B	C	
4.	A	B	D	
5.	A	C	D	
6.	A	B	C	D

The Seven Winning Groups

1.	A	B		
2.	A	C		
3.	A	B	C	
4.	A	B	D	
5.	A	C	D	
6.	A	B	C	D
7.	B	C	D	

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To find the Banzhaf Power index divide each number in the above list by 12.

Swing Votes Marked in RED

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Swing Votes Marked in RED

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2.	A	C

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Swing Votes Marked in RED

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Observe that even though B has 50% more votes than C, they both have the same power index.

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Winning all three **swing states** was a **necessary condition** for a Clinton win.