

5.1 AREA BETWEEN TWO CURVES

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1. CALC I QUIZ

What is a derivative?

- a limit
- a rate of change of two variables
- the slope of the tangent line
- all of the above

2. CALC I QUIZ

What is an integral?

- a limit
- an anti-derivative
- the area under a curve
- all of the above

3. AREA BETWEEN CURVES

The problem: Find the area between two curves $f(x)$ and $g(x)$.

$$\text{Area} = \int_a^b \text{top}(x) - \text{bottom}(x) dx$$

where a and b are intersection points.

4. IMPLEMENTATION

This method requires you to

- draw the two graphs
- determine where the functions intersect
- determine which function is on top
- set up the integral
- evaluate the integral

5. EXAMPLE 1

Find the area of the region bounded by $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

6. EXAMPLE 2

Find the area of the region bounded by $g(x) = 2 - x^2$ and the line $f(x) = x$.

7. EXAMPLE 3

Find the area of the region bounded by $f(x) = x^3 - 2x^2 + x - 1$ and $g(x) = -x^2 + 3x - 1$.

8. HORIZONTAL ORIENTATION

Given curves $x = f(y)$ and $x = g(y)$

$$\text{Area} = \int_c^d \text{right}(y) - \text{left}(y) dy$$

where (a, c) and (b, d) are intersection points.

9. EXAMPLE 4

Find the area of the region bounded by $x = 3 - y^2$ and the line $y = x - 1$.