

Authentic Task: Plan a Subdivision

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Construction Problem

A contractor builds two types of homes: the standard model and the deluxe model. The standard model requires one lot, \$12,000 capital, 150 labor-days to build, and is sold for a profit of \$2400. The deluxe model requires one lot, \$32,000 capital, 200 labor-days to build, and is sold for a profit of \$3400. The contractor has 150 lots. The bank is willing to loan him \$2,880,000 for the project and he has a maximum labor force available of 24,000 labor-days. How many houses should he build to realize the greatest profit?

Overview

We will show how to solve “The Construction Problem”

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- It is complicated.
- The numbers are unrealistic.

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Good points:

- It is a nice example of an authentic task.
- It helps you review concepts about lines and intercepts.

Bad points:

- It is complicated.
- The numbers are unrealistic.
- It is not a DIY problem. You will need guidance to obtain a solution.

Some Remarks

Some problems involve learning a precise method.

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- Knitting

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- Knitting
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Other problems are more suited to an independent approach.

- Games, such as Tic-tac-toe or Sudoku
- The cell phone tower problem

Organizing the Data

Resource	Standard	Deluxe	Available

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Resource	Standard	Deluxe	Available
Lot			

Organizing the Data

Resource	Standard	Deluxe	Available
Lot			
Capital			

Organizing the Data

Resource	Standard	Deluxe	Available
Lot			
Capital			
Labor			

Organizing the Data

Resource	Standard	Deluxe	Available
Lot			
Capital			
Labor			
Profit			

Organizing the Data

Resource	Standard	Deluxe	Available
Lot	1	1	
Capital			
Labor			
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Resource	Standard	Deluxe	Available
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Capital	12,000	32,000	
Labor			
Profit			

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Profit			

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Resource Inequalities:

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Resource Inequalities:

- Lot $1x + 1y \leq 150$

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Profit Equation:

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- Profit $P = 2400x + 3400y$

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- Resource Inequalities and Profit:

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 - **Lot** $x + y \leq 150$
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Graph the lot inequality

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$$x + y = 150$$

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How do we draw this line?

Graphing the line

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The x –intercept is the point where the line crosses the x –axis, or when $y = 0$.

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Plugging $y = 0$ into the equation $x + y = 150$, we see $x = 150$

So the x -intercept is the point $(150, 0)$.

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Plugging $y = 0$ into the equation $x + y = 150$, we see $x = 150$

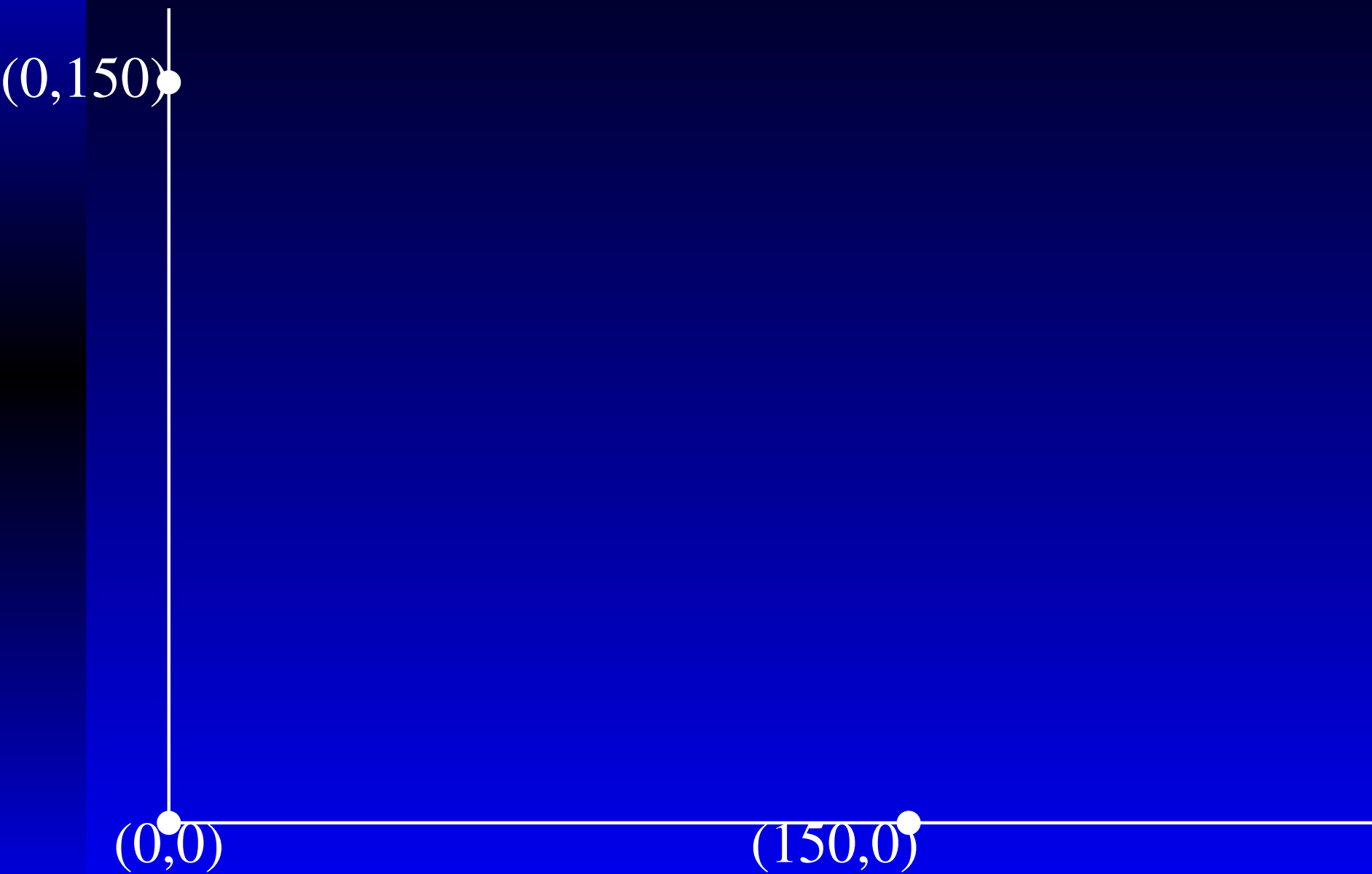
So the x -intercept is the point $(150, 0)$.

Similarly the y -intercept is the point $(0, 150)$.

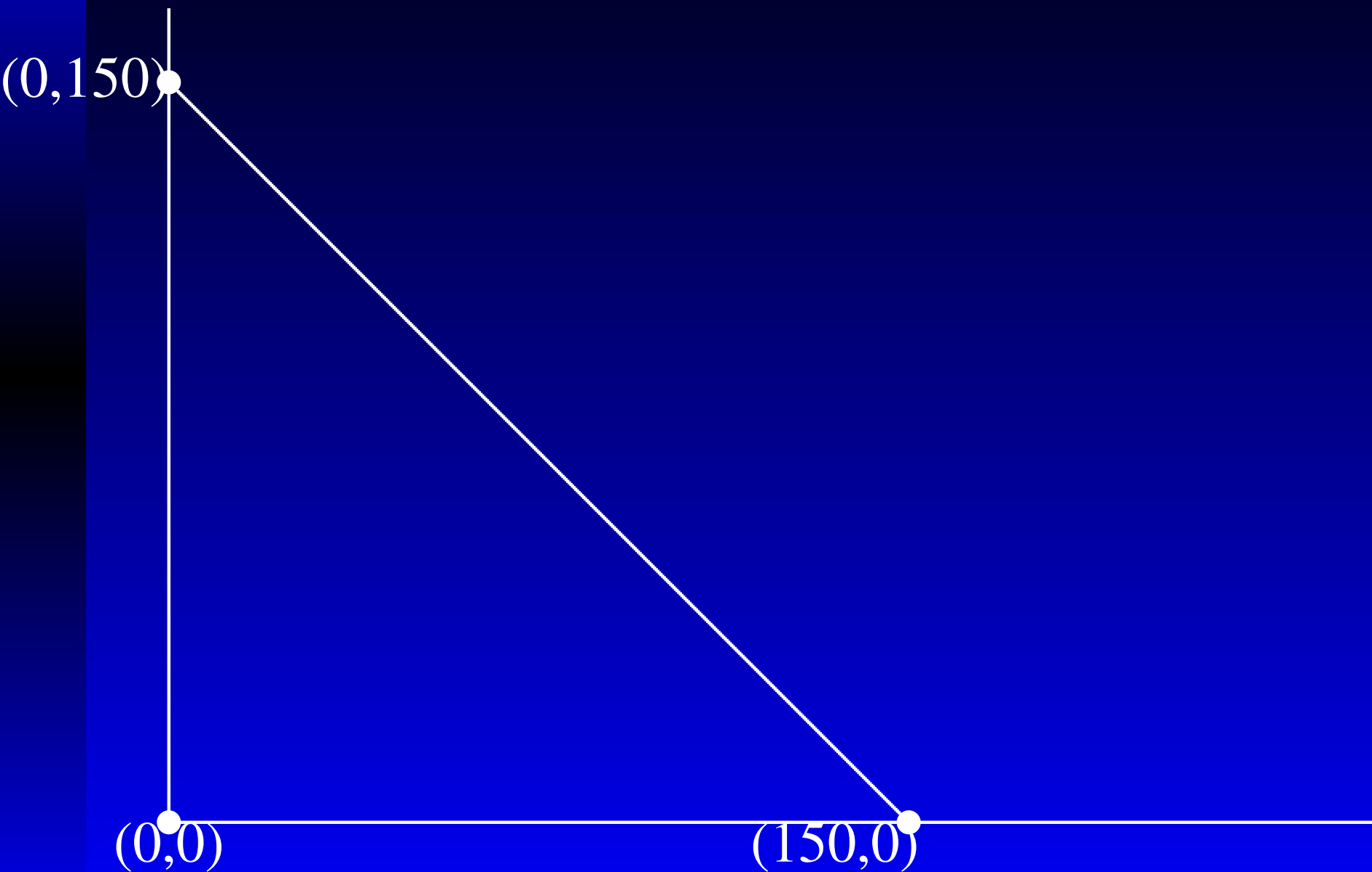
Graph $x + y = 150$



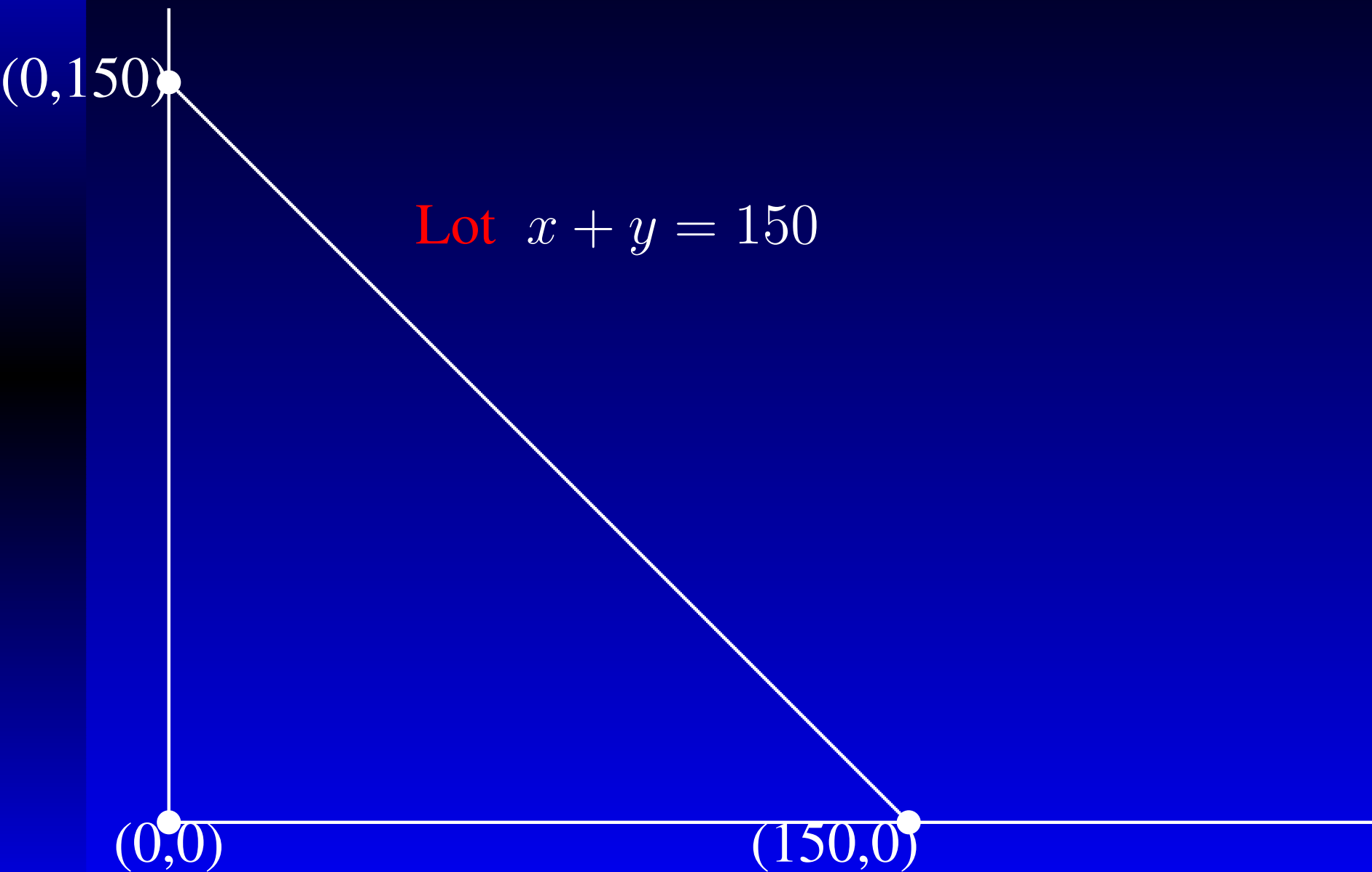
Graph $x + y = 150$



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DIY: Graph the capital line

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Capital

$$3x + 8y = 720$$

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Capital $3x + 8y = 720$

by finding the x - and y -intercepts.

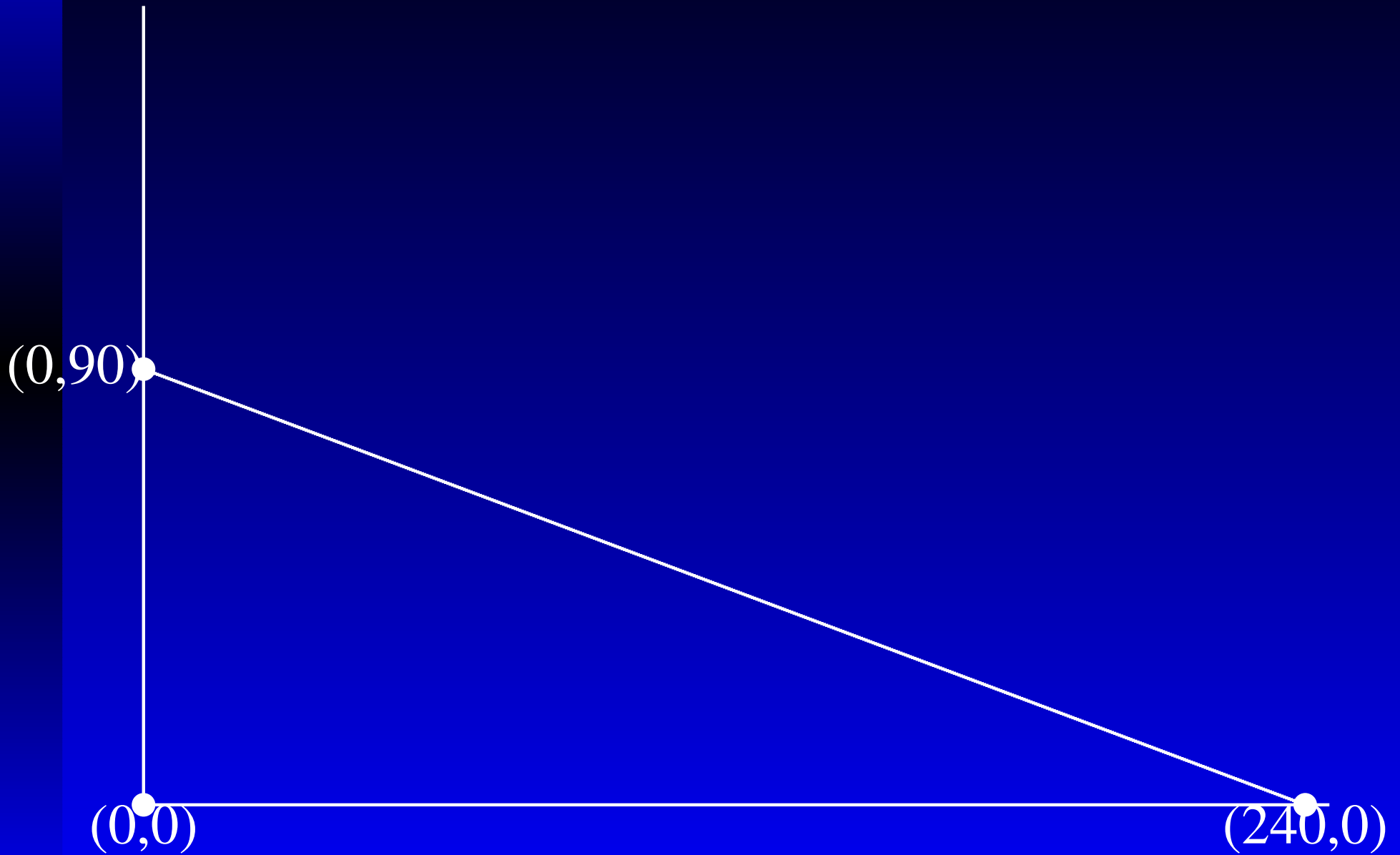
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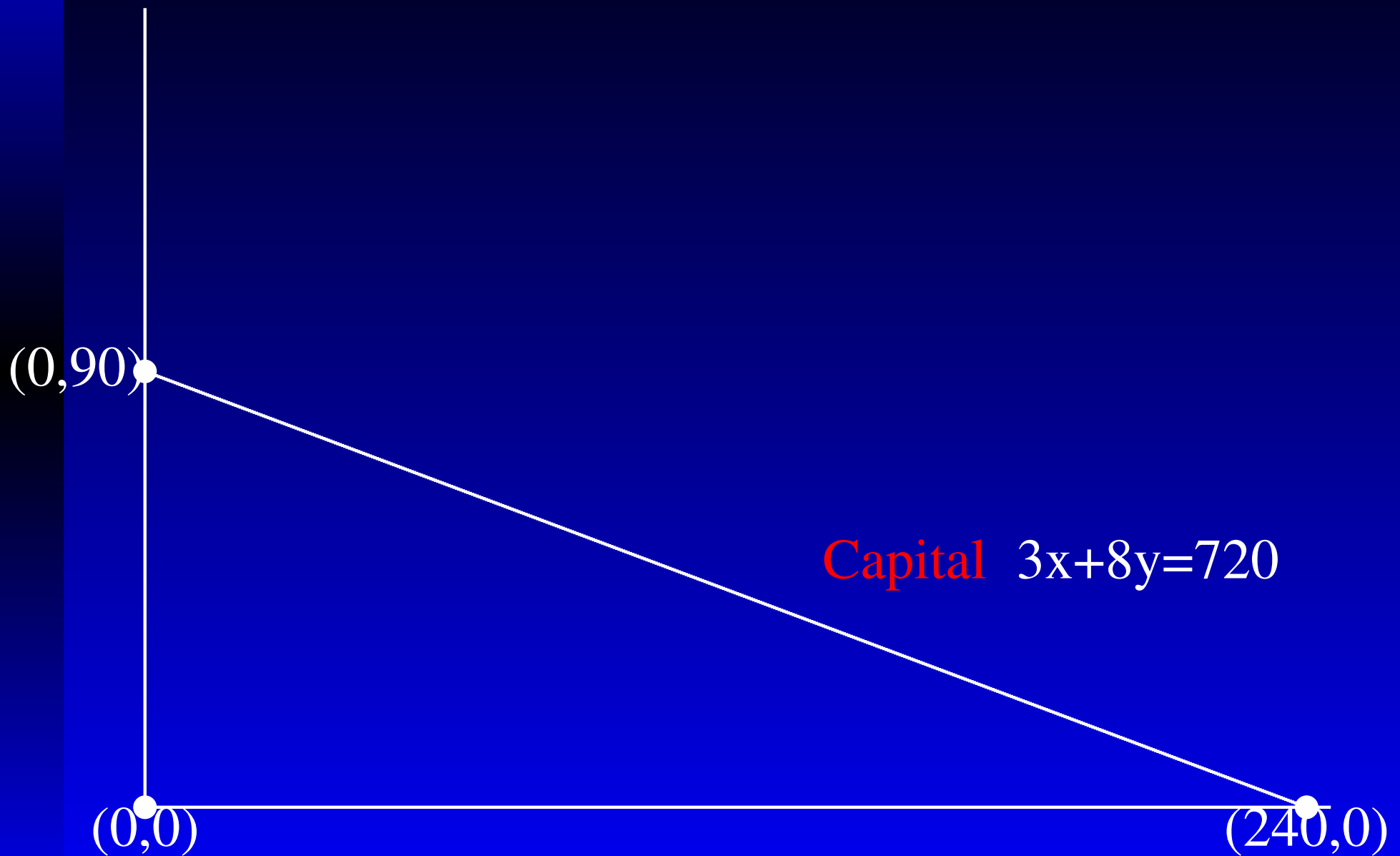
Graph $3x + 8y = 720$



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Graph the labor line

Graph

Graph the labor line

Graph

Labor

$$3x + 4y = 480$$

Graph the labor line

Graph

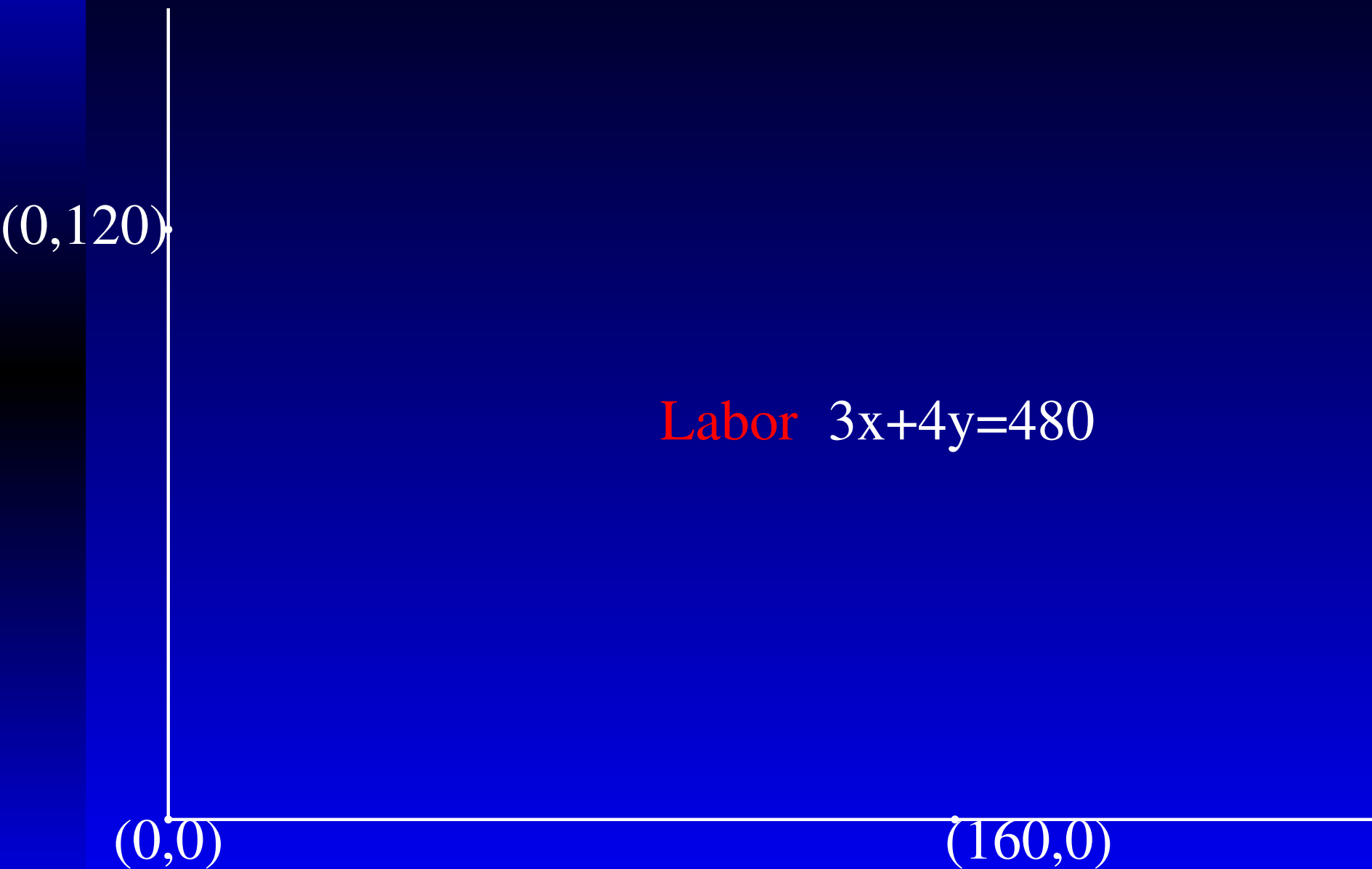
Labor $3x + 4y = 480$

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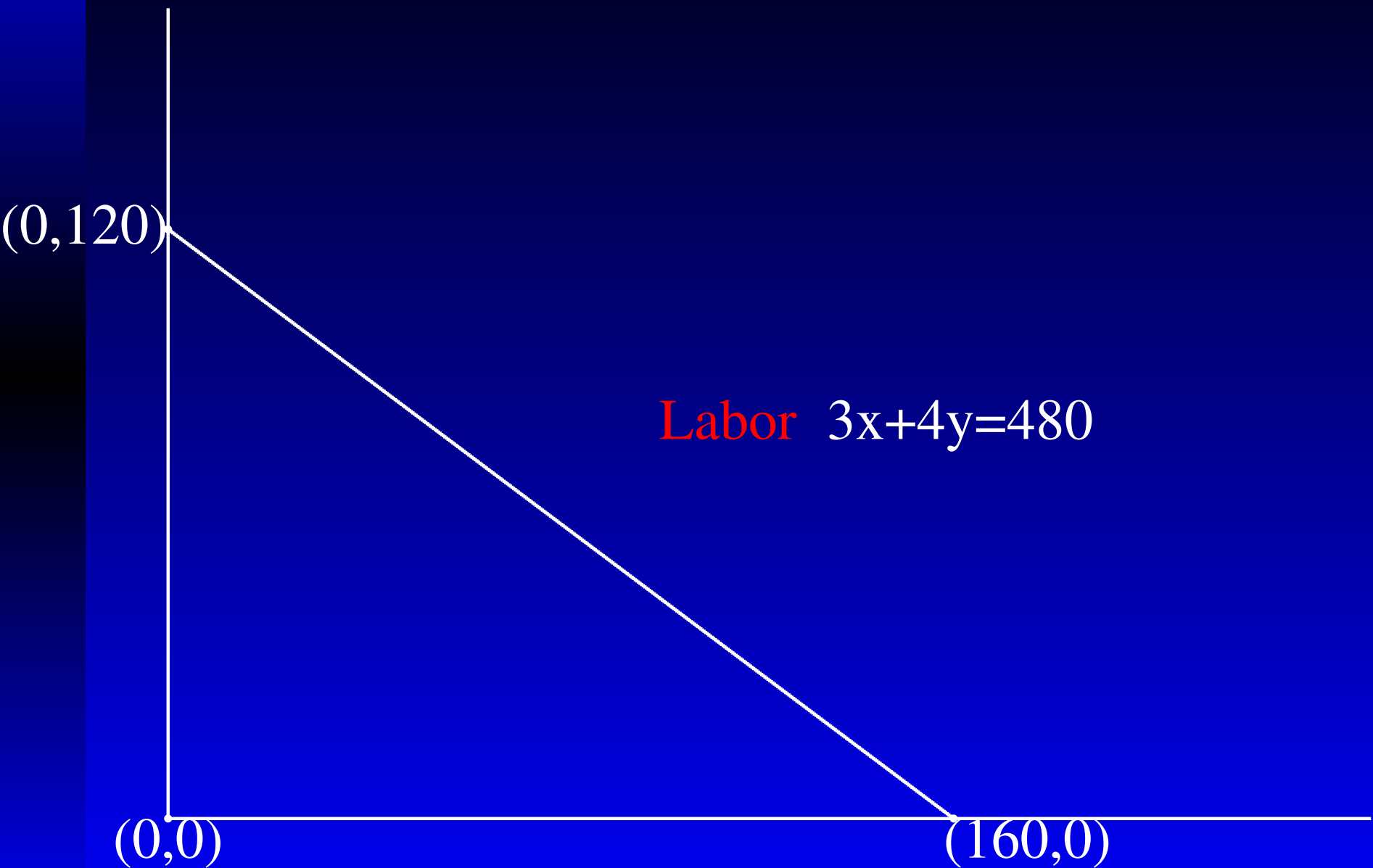
Graph $3x + 4y = 480$



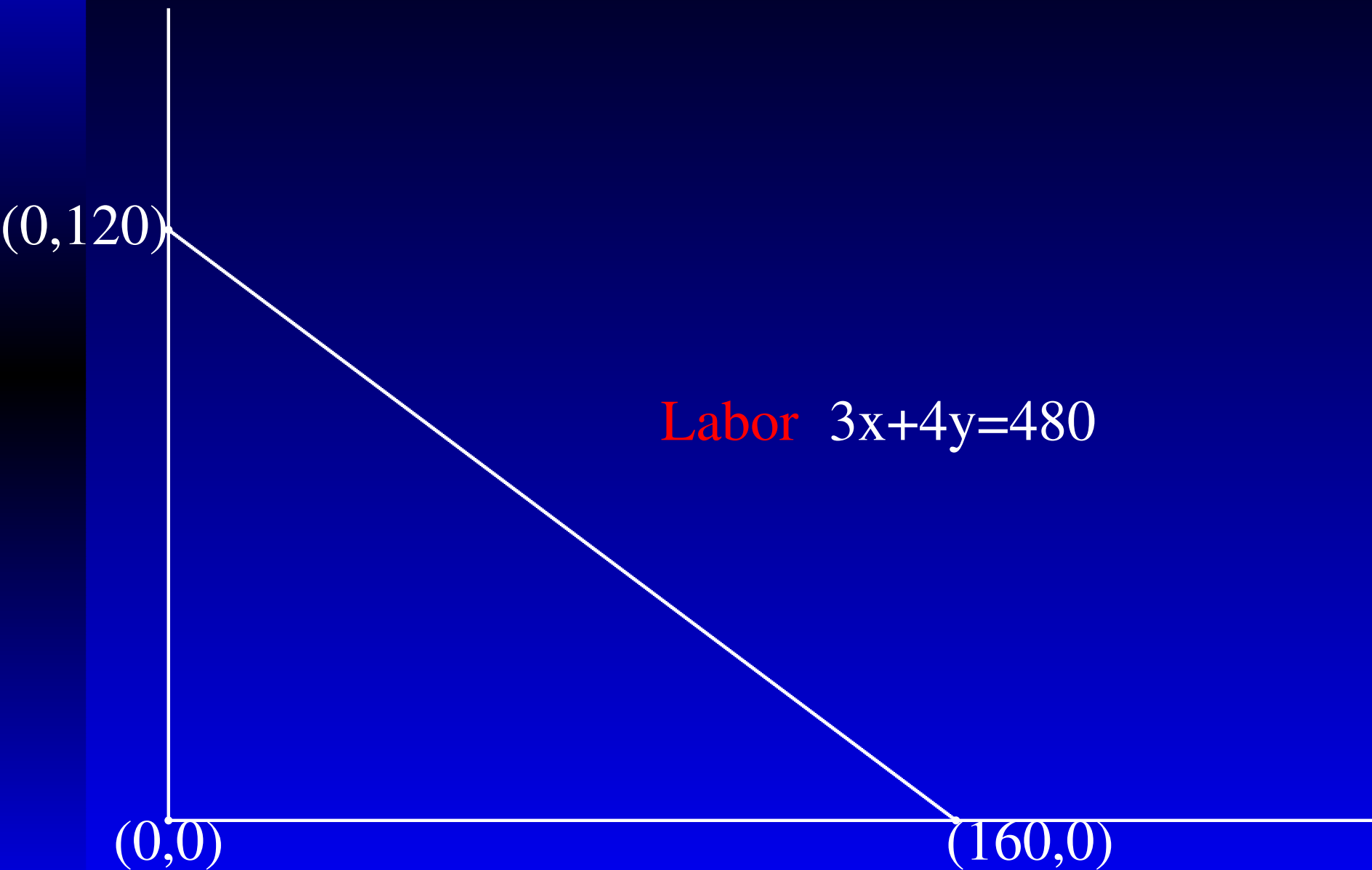
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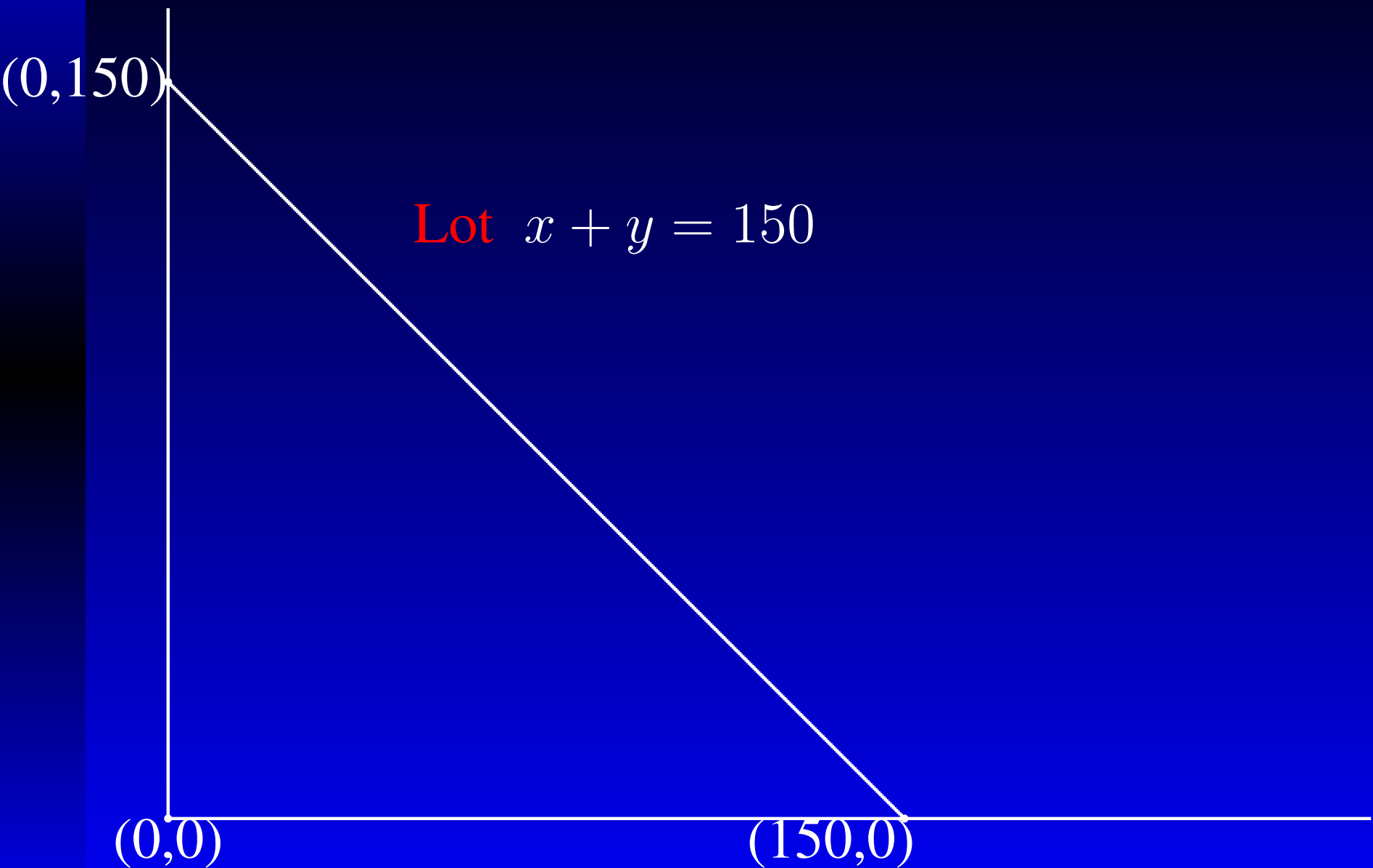
Graph $3x + 4y = 480$



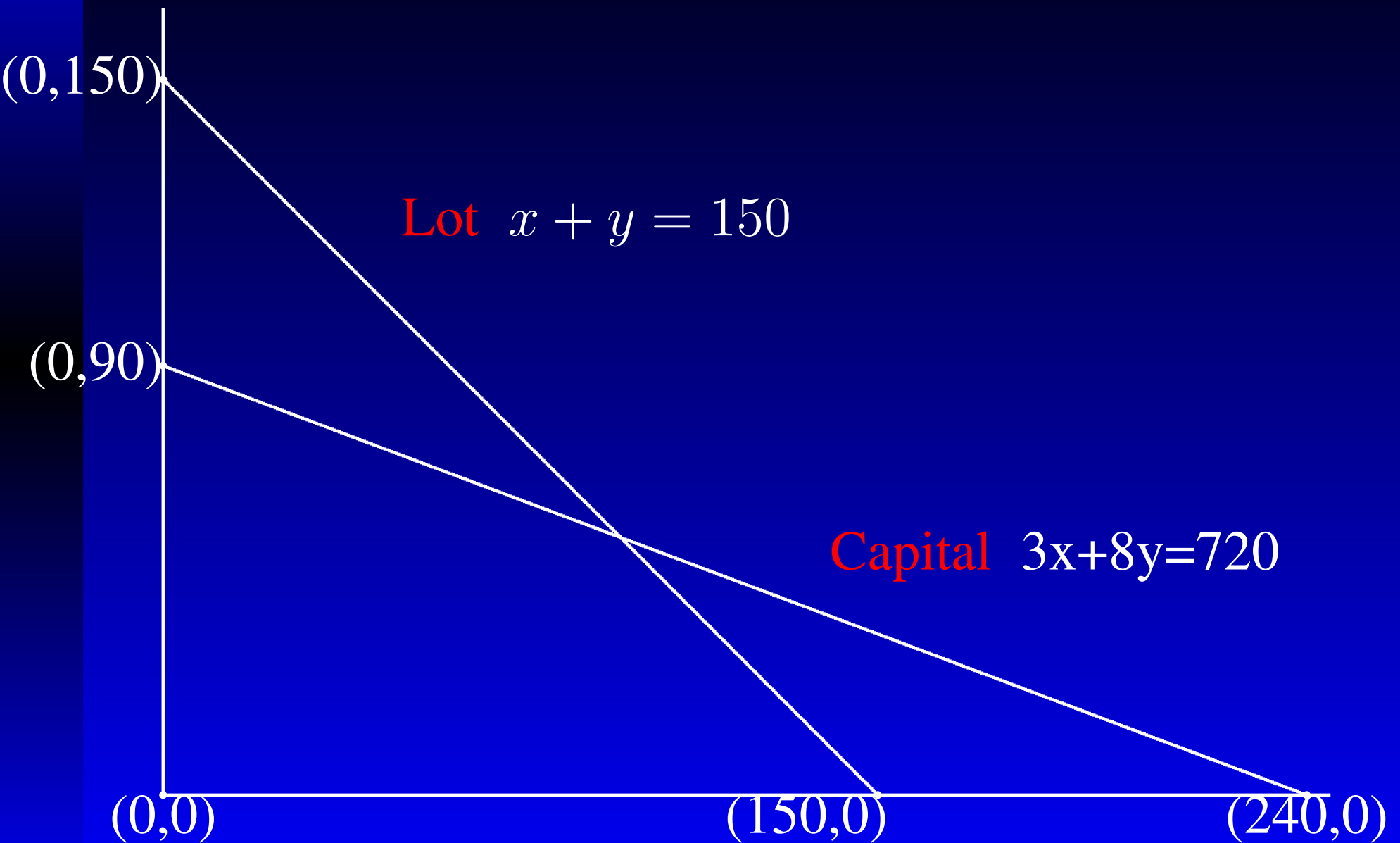
Complete Graph



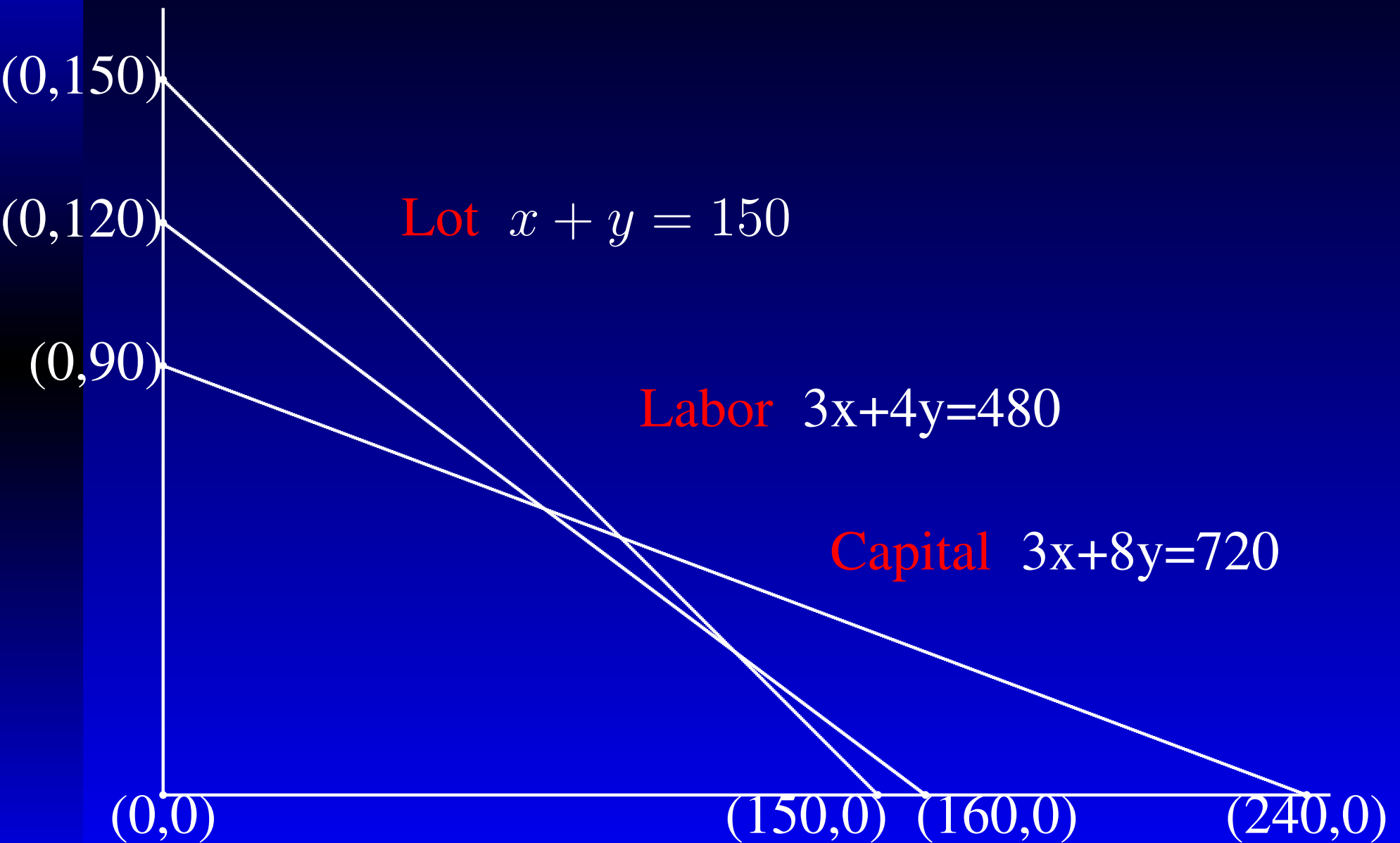
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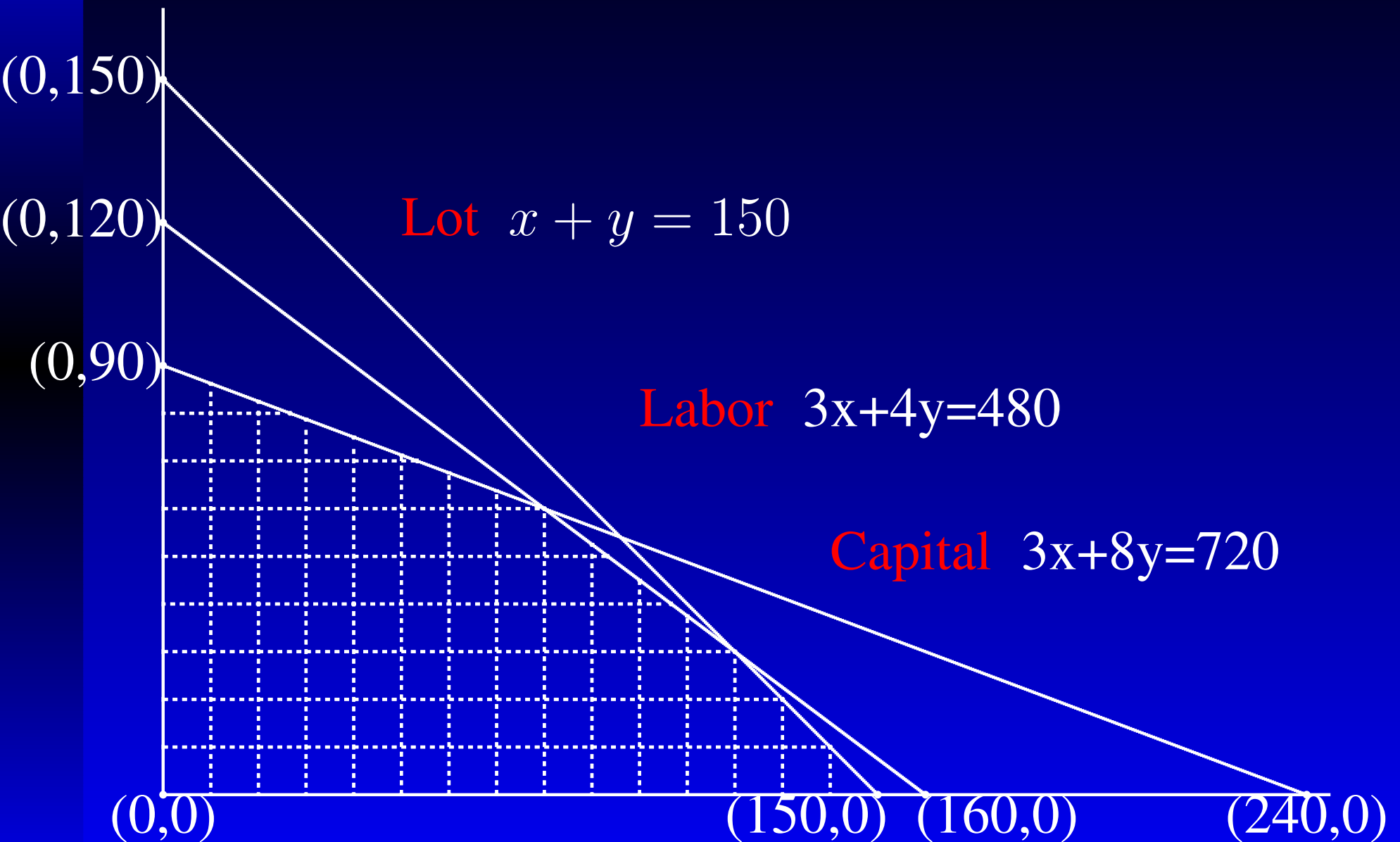
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Solving Linear Programming Problems

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Linear Programming Rule:

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Linear Programming Rule:

Maximums (and **minimums**) occur at corner points

Where are the corner points?

Looking at the complete graph, we see there are five corner points.

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Three of them are on the graph:

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The other two corner points are the intersection of the capital and labor lines and the intersection of the lot and labor lines.

Where are the corner points?

Looking at the complete graph, we see there are five corner points.

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The other two corner points are the intersection of the capital and labor lines and the intersection of the lot and labor lines.

What about the intersection of the lot and capital lines?

DIY: Where do Capital and Labor Lines Cross?

Find the intersection of the lines:

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Find the intersection of the lines:

$$\text{Capital} \quad 3x + 8y = 720$$

$$\text{Labor} \quad 3x + 4y = 480$$

Where do Capital and Labor Lines Cross?

- Write the equations:

Where do Capital and Labor Lines Cross?

- Write the equations:

$$\text{Capital} \quad 3x + 8y = 720$$

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Subtract} \quad 4y = 240$$

Where do Capital and Labor Lines Cross?

- Write the equations:

$$\text{Capital} \quad 3x + 8y = 720$$

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Subtract} \quad 4y = 240$$

- Divide by 4:

Where do Capital and Labor Lines Cross?

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- Divide by 4:

- So $y = \frac{240}{4} = 60$

Where do Capital and Labor Lines Cross?

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$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Subtract} \quad 4y = 240$$

- Divide by 4:

- So $y = \frac{240}{4} = 60$

- Plug $y = 60$ back into the Labor equation
 $3x + 4y = 480$

Capital and Labor Line Intersection

$$3x + 4 \cdot 60 = 480$$

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$$3x + 240 = 480$$

Capital and Labor Line Intersection

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$$3x = 480 - 240 = 240$$

Capital and Labor Line Intersection

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$$3x + 240 = 480$$

$$3x = 480 - 240 = 240$$

$$x = \frac{240}{3} = 80$$

Capital and Labor Line Intersection

$$3x + 4 \cdot 60 = 480$$

$$3x + 240 = 480$$

$$3x = 480 - 240 = 240$$

$$x = \frac{240}{3} = 80$$

- The intersection point is (80, 60).

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Find the intersection of the lines:

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Lot} \quad x + y = 150$$

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- Write the equations:

Labor $3x + 4y = 480$

Lot $x + y = 150$

Where do Labor and Lot Lines Intersect?

- Write the equations:
Labor $3x + 4y = 480$
Lot $x + y = 150$
- Multiply Lot equation by 3

Where do Labor and Lot Lines Intersect?

- Write the equations:

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Lot} \quad x + y = 150$$

- Multiply Lot equation by 3

$$\text{Capital} \quad 3x + 4y = 480$$

$$\text{Lot} \quad 3x + 3y = 450$$

$$\text{Subtract} \quad y = 30$$

Where do Labor and Lot Lines Intersect?

- Write the equations:

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Lot} \quad x + y = 150$$

- Multiply Lot equation by 3

$$\text{Capital} \quad 3x + 4y = 480$$

$$\text{Lot} \quad 3x + 3y = 450$$

$$\text{Subtract} \quad y = 30$$

- Plug $y = 30$ back into the Lot equation

Where do Labor and Lot Lines Intersect?

- Write the equations:

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Lot} \quad x + y = 150$$

- Multiply Lot equation by 3

$$\text{Capital} \quad 3x + 4y = 480$$

$$\text{Lot} \quad 3x + 3y = 450$$

$$\text{Subtract} \quad y = 30$$

- Plug $y = 30$ back into the Lot equation
- $x + 30 = 150$ or $x = 120$

Where do Labor and Lot Lines Intersect?

- Write the equations:

$$\text{Labor} \quad 3x + 4y = 480$$

$$\text{Lot} \quad x + y = 150$$

- Multiply Lot equation by 3

$$\text{Capital} \quad 3x + 4y = 480$$

$$\text{Lot} \quad 3x + 3y = 450$$

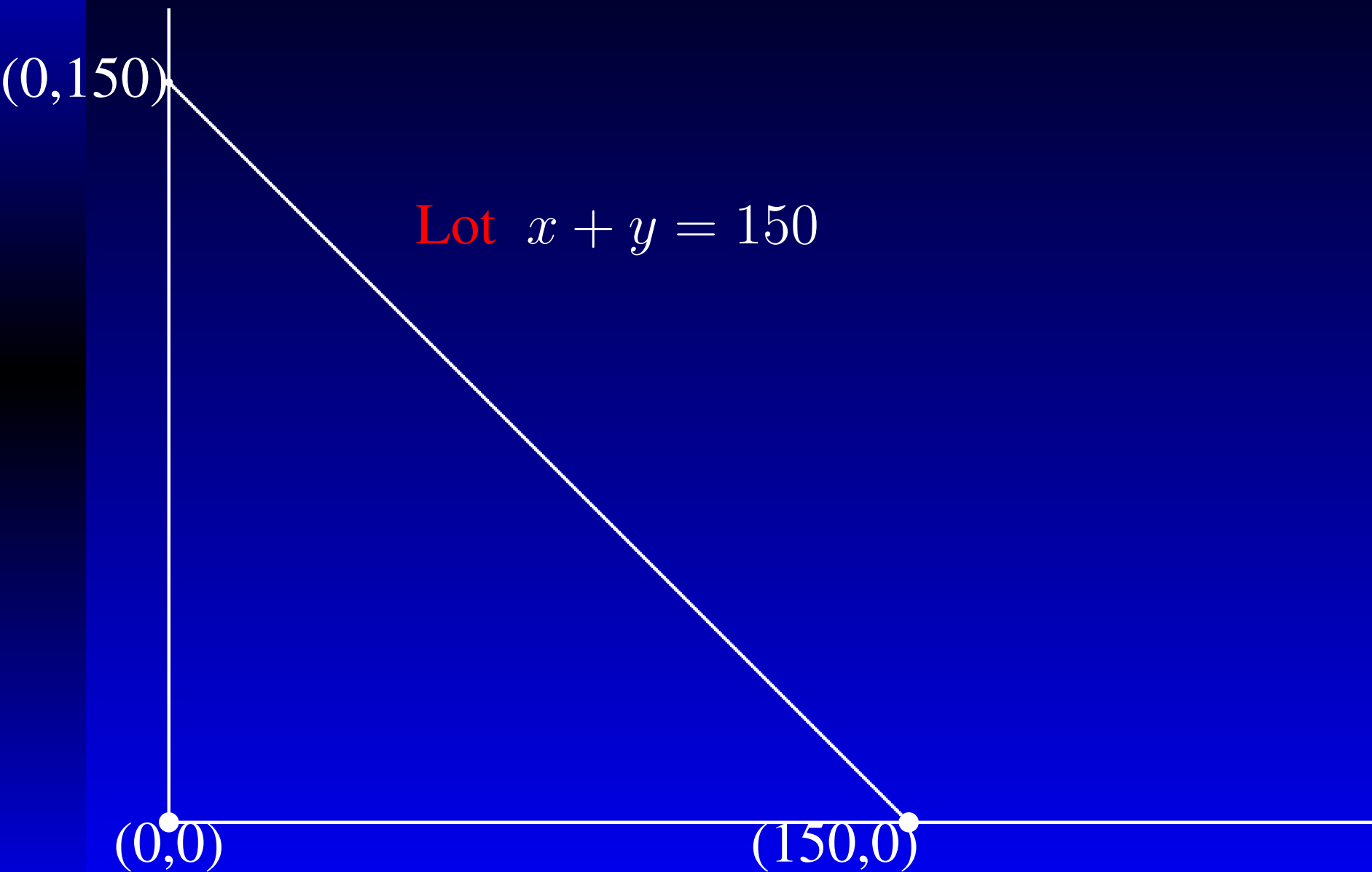
$$\text{Subtract} \quad y = 30$$

- Plug $y = 30$ back into the Lot equation
- $x + 30 = 150$ or $x = 120$
- The intersection point is $(120, 30)$

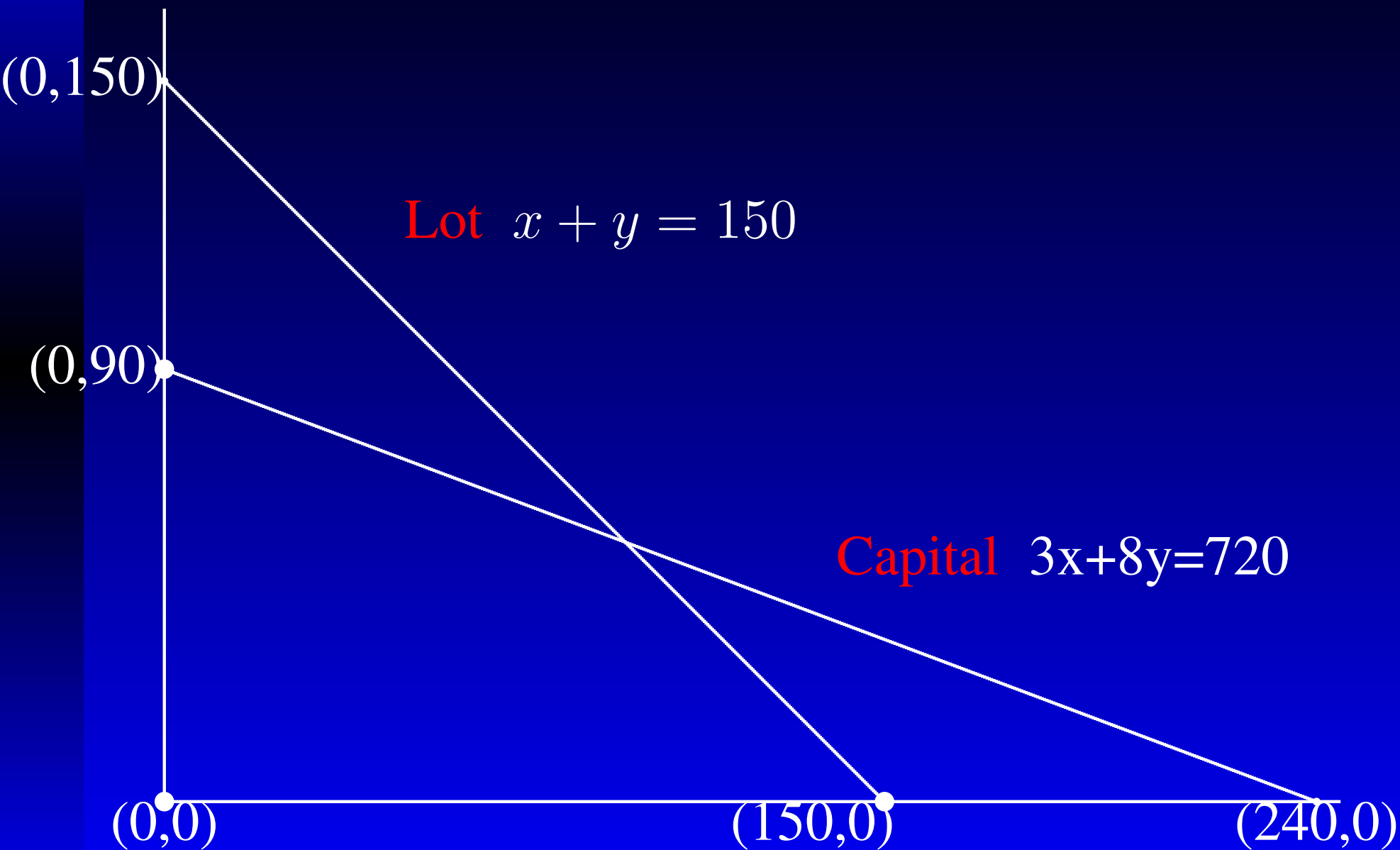
Graph



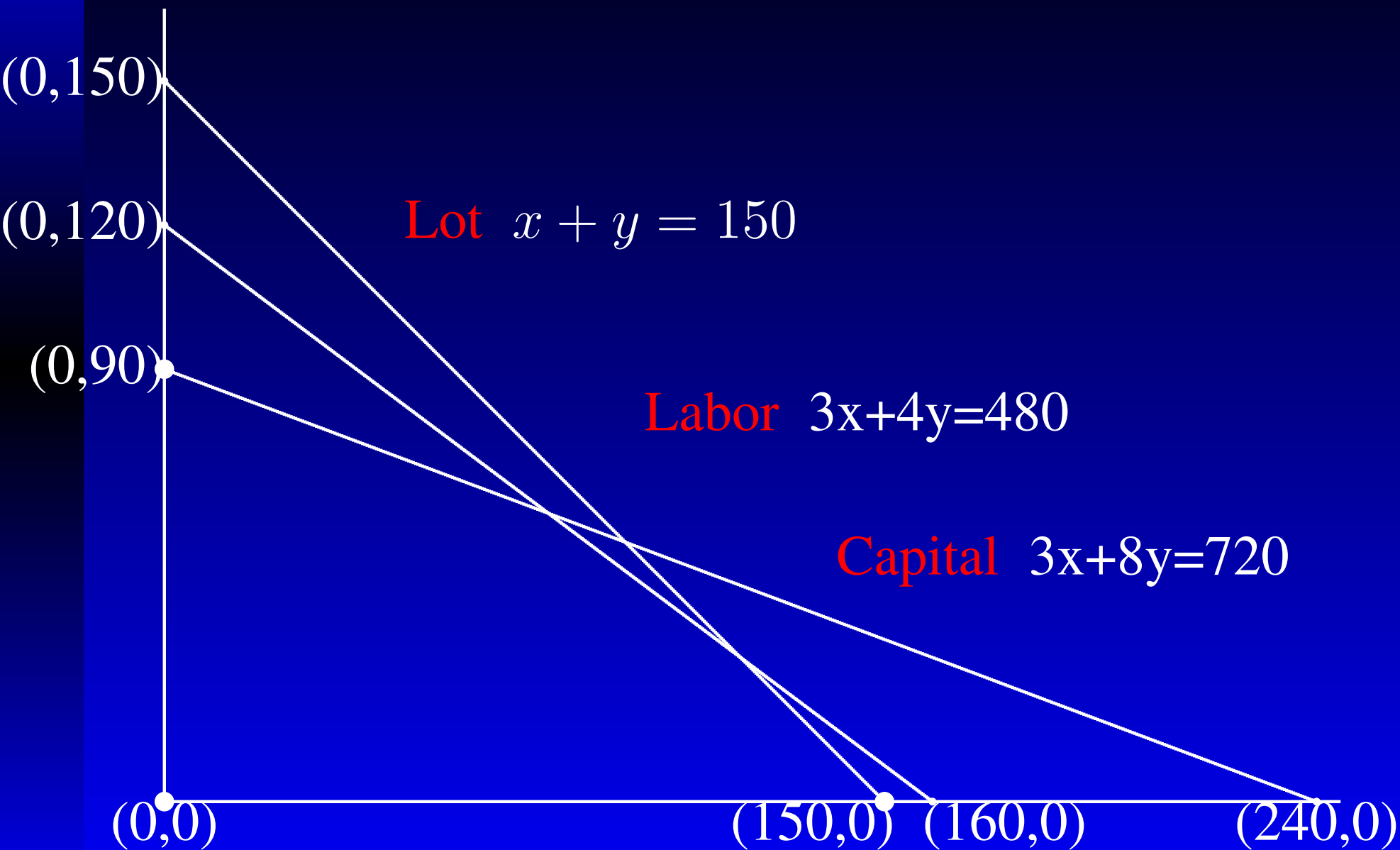
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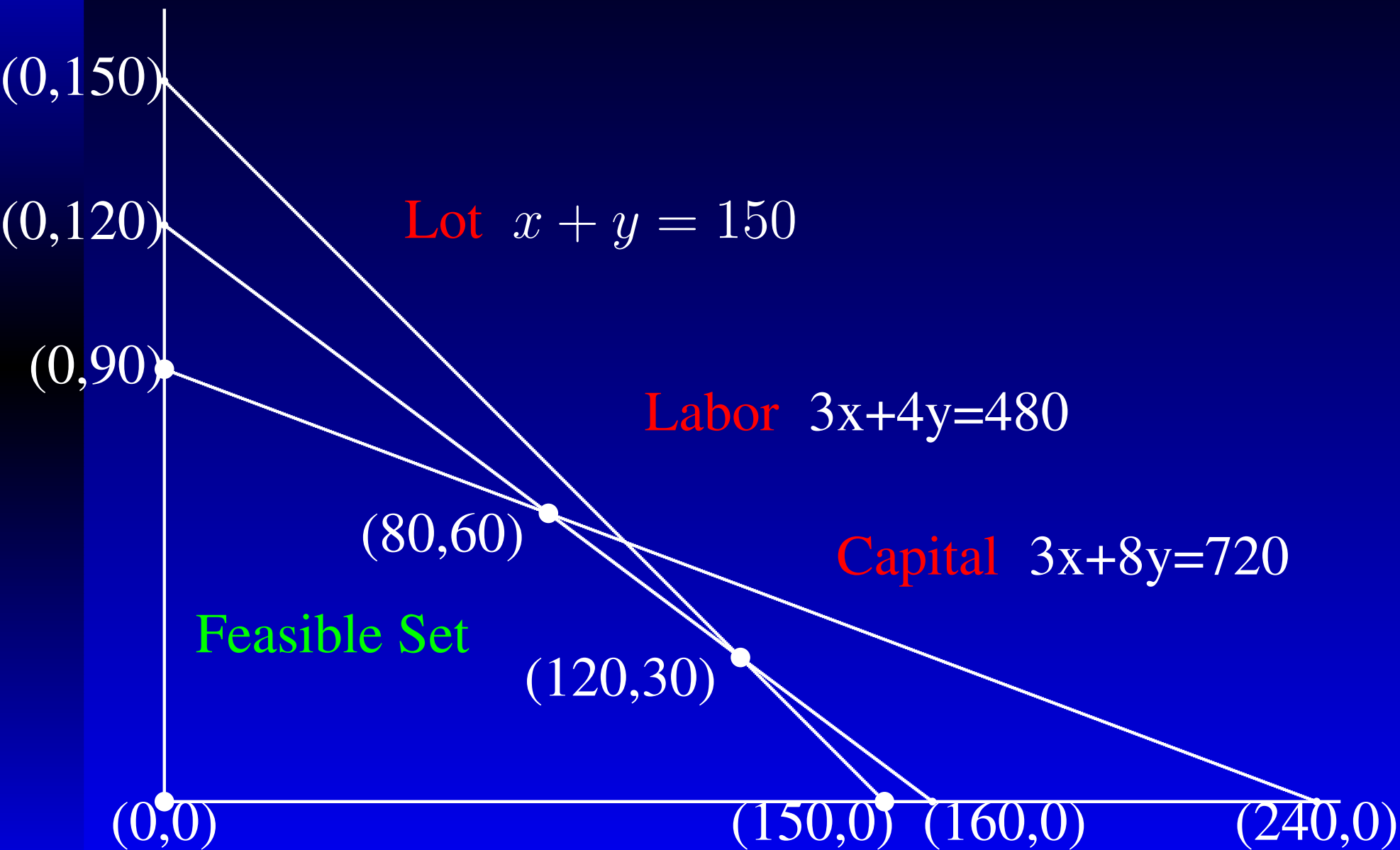
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Graph



Graph



Solving the Problem

x	y	$P = 2400x + 3400y$

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	
0	90	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	
0	90	
80	60	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	
0	90	
80	60	
120	30	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	
0	90	
80	60	
120	30	
150	0	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	
80	60	
120	30	
150	0	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	
120	30	
150	0	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	
150	0	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	\$360,000

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	\$360,000

Maximum Profit

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	\$360,000

Maximum Profit

- 80 standard

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	\$360,000

Maximum Profit

- 80 standard
- 60 deluxe

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	\$360,000

Maximum Profit

- 80 standard
- 60 deluxe
- \$396,000 profit

Solving the Problem

x	y	$P = 2400x + 3400y$
0	0	0
0	90	\$306,000
80	60	\$396,000
120	30	\$390,000
150	0	\$360,000

Maximum Profit

- 80 standard
- 60 deluxe
- \$396,000 profit
- Note: 10 lots are unused