

MATH 229 LECTURE NOTES:

CHAPTER 2.1 DERIVATIVES

1. DEFINITION OF DERIVATIVE

The **derivative** of the function $f(x)$ is defined to be

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The prime in the symbol $f'(x)$ signifies the derivative of the function $f(x)$

Read $f'(x)$ as “the derivative of f at x ” or as “ f -prime of x ”

Sometimes $f'(x)$ is called the **derived** function

2. SOME OBSERVATIONS

The definition

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

requires us to

- find $f(x+h)$
- find the difference $f(x+h) - f(x)$
- find the quotient $\frac{f(x+h) - f(x)}{h}$
- evaluate the limit of this quotient as $h \rightarrow 0$

3. EXAMPLE 1.

Compute the derivative $f'(8)$ for the function

$$f(x) = 3x^2 - 4x + 1.$$

$$\begin{aligned} f'(8) &= \lim_{h \rightarrow 0} \frac{f(8+h) - f(8)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3(8+h)^2 - 4(8+h) + 1) - (3 \cdot 8^2 - 4 \cdot 8 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3(64 + 16h + h^2) - 32 - 4h + 1) - 161}{h} \\ &= \lim_{h \rightarrow 0} \frac{44h + 3h^2}{h} = \lim_{h \rightarrow 0} \frac{(44 + 3h)h}{h} \\ &= \lim_{h \rightarrow 0} 44 + 3h = 44 \end{aligned}$$

4. SO WHAT

What information do we get from knowing that the derivative of the function $f(x) = 3x^2 - 4x + 1$ at $x = 8$ has the value 44?

What does the 44 tell us?

Answer 1. Relative Rate of Change.

It tells us that when $x = 8$, the y -values are increasing exactly 44 times as fast as the x -values.

Answer 2. Tangent Line

It tells us that on the graph of $y = f(x)$, the **tangent line** to the curve at the point $(8, 161)$ has a slope of exactly 44.

Answer 3. Information about the graph of $y = f(x)$. Since the y -values are increasing, the curve must be going upward at the point $(8, 161)$

5. EXAMPLE 2.

Compute the derivative $f'(3)$ for $f(x) = \frac{1}{x+1}$.

$$\begin{aligned}
 f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{3+h+1} - \frac{1}{4}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{4+h} - \frac{1}{4} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{4+h} \frac{4}{4} - \frac{1}{4} \frac{4+h}{4+h} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{4}{4(4+h)} - \frac{4+h}{4(4+h)} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \frac{4 - (4+h)}{4(4+h)} = \lim_{h \rightarrow 0} \frac{1}{h} \frac{-h}{4(4+h)} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{4(4+h)} = -\frac{1}{16}
 \end{aligned}$$

6. EXAMPLE 3.

Compute the derivative $f'(9)$ for the function

$$f(x) = \sqrt{x}.$$

$$\begin{aligned} f'(9) &= \lim_{h \rightarrow 0} \frac{f(9+h) - f(9)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h} \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3} \\ &= \lim_{h \rightarrow 0} \frac{(9+h) - 9}{h} \frac{1}{\sqrt{9+h} + 3} \\ &= \lim_{h \rightarrow 0} \frac{h}{h} \frac{1}{\sqrt{9+h} + 3} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{9+h} + 3} = \frac{1}{6} \end{aligned}$$