

To Infinity and Beyond

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Proof that $1 = 2$

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What went wrong?

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**YOU CANNOT
DIVIDE BY ZERO!!!**

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Question: What would you say to this student?

Some observations

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Question: What would you say to this student?

Different ways to write fractions

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In this last notation, the fraction $\frac{a}{b}$ is written as the ordered pair (a, b) .

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How do we know when two fractions are the same ?

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For example, $(14, 21) = (2, 3)$.

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Remember: (a, b) denotes the fraction $\frac{a}{b}$.

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Make up your own example.

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according to our formula for equivalent fractions

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Why ?

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This just means the point lines on the y -axis.

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This looks very similar to our analysis for $(0, 1)$.

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It's gotta go.

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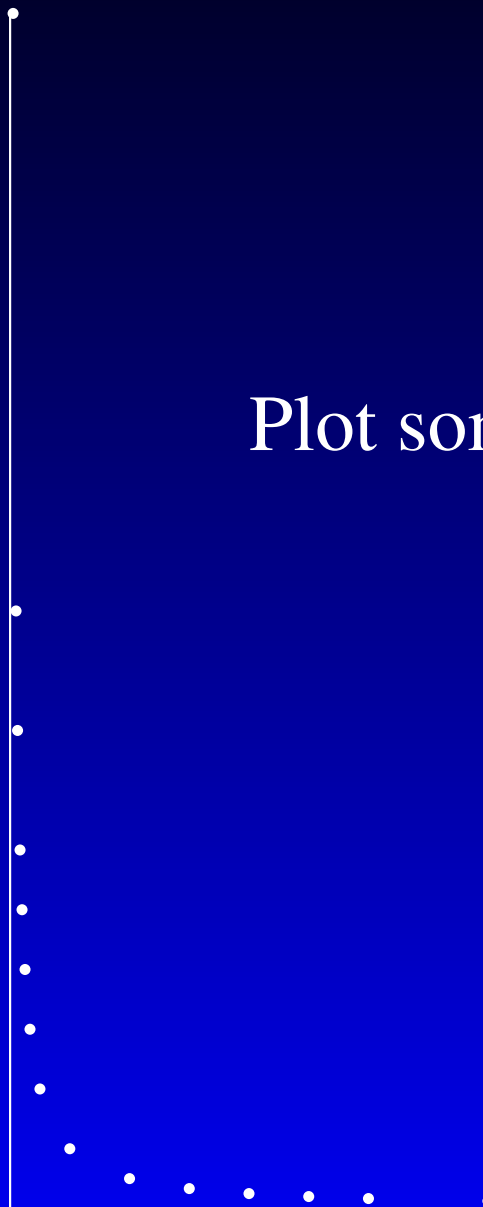
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That is, 0 has no reciprocal.

Graph of $y = \frac{1}{x}$



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Plot some points.

Graph of $y = \frac{1}{x}$

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$(\frac{1}{10}, 10)$

$(\frac{1}{4}, 4)$

$(\frac{1}{2}, 2)$

$(1, 1)$

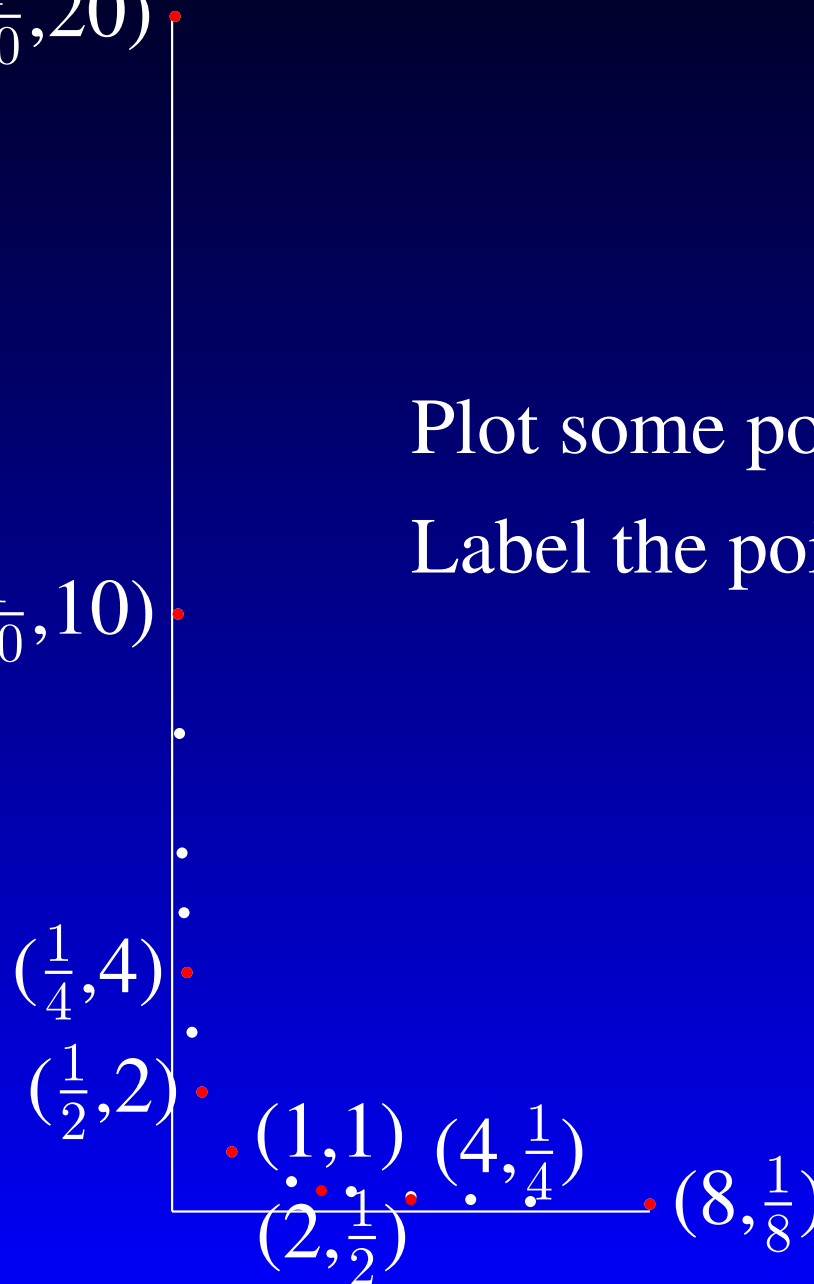
$(2, \frac{1}{2})$

$(4, \frac{1}{4})$

$(8, \frac{1}{8})$

Plot some points.

Label the points.



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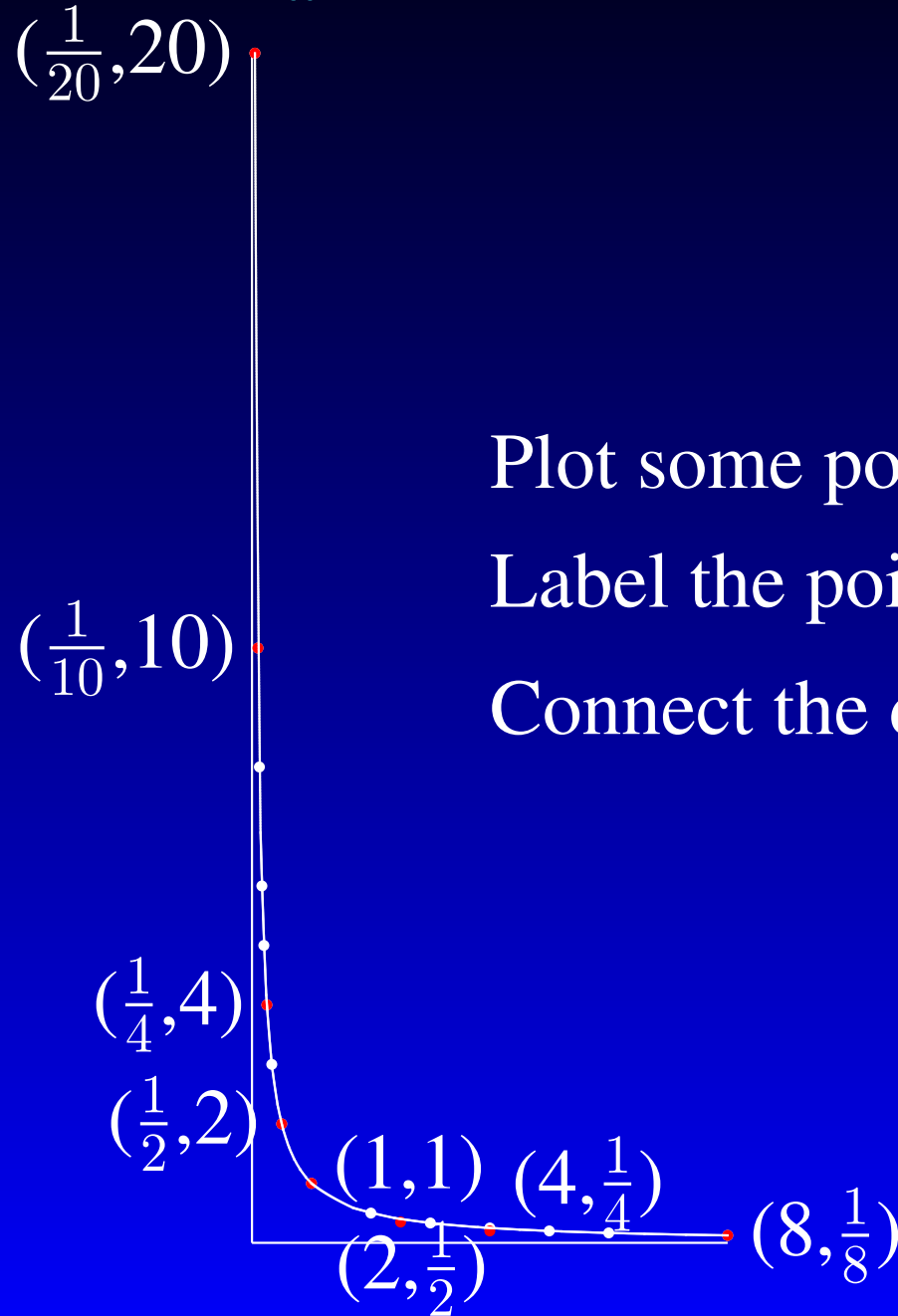
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Plot some points.

Label the points.

Connect the dots.



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Mathematicians say that $\frac{1}{x}$ increases without bound

or that $\frac{1}{x}$ approaches infinity.

Introducing ∞

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The equation $(5, 1) \times (0, 1) = (0, 1)$

Introducing ∞

Some mathematicians allow the expression $(1, 0)$. They call it “the point at infinity” or just ∞ , for short. Although $(1, 0)$ is not a legitimate fraction, the arithmetic works provided we do not multiply it by $(0, 1)$. The equation $(5, 1) \times (0, 1) = (0, 1)$ just says 5 times infinity is still infinity.

Reflections about Infinity

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There is a car company called Infiniti.

There will never be a car company called Square Root or Logarithm.