

ACTIVITY 12. FIBONACCI NUMBERS

1. The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

2. The Fibonacci Recurrence

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$$

3. Fibonacci's Party by Cynthia Ellis

The great mathematician Fibonacci
Went out to the market and bought a Hibachi
He decided to give a small Bar B Que
For himself, his wife, and a good friend or two.

So he called his friend Joe and he asked him to come
With a small jug of wine or a bottle of rum.
"My wife (one) and I (one) make two" figured he,
"And with Joseph attending the total is three."

But then the telephone rang in the hall:
His parents would be there, making five guests in all.
And his wife told him also her parents were coming.
With sister Loretta—now eight was his summing.

But, oh, he'd forgotten Joe's girlfriend Eileen.
With her and her family the total's thireteen.
And Loretta brings friends to wherever there's fun.
So he counted it up and he got twenty-one.

Just then he remembered the neighbors next door.
They'd certainly be there to make thirty-four.
But then his club's football team pulled in the drive.
And he tore at his hair as he thought "Fifty-five!"

While out in the street he saw line after line
Of neighborhood moochers to make eighty-nine.
And 'round from the alley there came at a trot
His boss and co-workers, the whole bloomin' lot.

Fib went to the gameroom and sat on the floor
And figured the total as one-forty-four.
So he crawled to the bar and swallowed a dose
And started to wonder how three grew to gross.

He pulled out his list and he started to count,
Carefully writing down every amount.
And discovered the sequence that now bears his name,
Thanks to the party where everyone came.

4. Table 1. factorization of the first 30 Fibonacci numbers into primes

n	F_n	factors	n	F_n	prime factors	n	F_n	prime factors
1	1		11	89	89	21	10946	$2 \cdot 13 \cdot 421$
2	1		12	144	$2^4 \cdot 3^2$	22	17711	$89 \cdot 199$
3	2	2	13	233	233	23	28657	prime
4	3	3	14	377	$13 \cdot 29$	24	46368	$2^5 \cdot 3^2 \cdot 7 \cdot 23$
5	5	5	15	610	$2 \cdot 5 \cdot 61$	25	75025	$5^2 \cdot 3001$
6	8	2^3	16	987	$3 \cdot 7 \cdot 47$	26	121393	$233 \cdot 521$
7	13	13	17	1597	prime	27	196418	$2 \cdot 17 \cdot 53 \cdot 109$
8	21	$3 \cdot 7$	18	2584	$2^3 \cdot 17 \cdot 19$	28	317811	$3 \cdot 13 \cdot 29 \cdot 281$
9	34	$2 \cdot 17$	19	4181	$37 \cdot 113$	29	514229	prime
10	55	$5 \cdot 11$	20	6765	$3 \cdot 5 \cdot 11 \cdot 41$	30	832040	$2^3 \cdot 5 \cdot 11 \cdot 31 \cdot 61$

5. Distribution of prime divisors of Fibonacci Numbers

(a) For each prime in the table below, fill in all values of n (up to $n = 30$) for which p divides F_n .

p	$n : p \text{ divides } F_n$
2	
3	
5	
7	
11	10, 20, 30
13	7, 14, 21, 28
17	
19	
23	
29	
31	

p	$n : p \text{ divides } F_n$
37	
41	
43	
47	
53	
61	
89	
109	
113	
199	
233	

(b) What can you say about the numbers in each row of the table?

(c) For each prime in the table, let F_k be the *first* Fibonacci number which is divisible by p . In the third column write the value of $p \bmod k$. If $p \bmod k = p - 1$, simply write “ -1 .” After all, -1 on a general p -clock is the same as $p - 1$. (One hour before midnight is 11 o'clock.)

For example, the prime $p = 11$ divides F_k for the first time when $k = 10$ and $11 \bmod 10 = 1$. The prime $p = 13$ first divides F_k when $k = 7$ and $13 \bmod 7 = -1$. The entries for $p = 11$ and 13 have been filled in for you.

p	k	$p \bmod k$
2		
3		
5		
7		
11	10	1
13	7	-1
17		

p	k	$p \bmod k$
19		
23		
29		
31		
37		
41		
43		

p	k	$p \bmod k$
47		
53		
61		
89		
109		
113		
199		

(d) What can you say about the values in the third column? Which prime stands out as an exception?

(e) The primes (outside of the one exception noted above) fall into two categories:

Type I primes: p divides F_{p-1}

Type II primes: p divides F_{p+1} .

For example, $p = 11$ is a Type I prime because 11 divides F_{10} , while $p = 13$ is a Type II prime because 13 divides F_{14} . Try to discover a rule for deciding which primes fall into each type. You may assume that once you know the value of the first k for which p divides F_k , then p divides F_n for $n = k, 2k, 3k$, etc. For example, since 41 divides F_{20} , we may assume 41 divides F_{40} , even though $n = 40$ is not in Table 1.

Hint: look at the last digit of the primes.