

1. (10 pts.) Let $f(x) = \sqrt{3x+5}$. Find $f'(x)$ using the limit definition of derivative

2. (15 pts.) Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\sin 4x}{3x}$ (b) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 4x + 3}$ (c) $\lim_{x \rightarrow \infty} \frac{5x^2 + 2x - 1}{7x^2 + 5x - 4}$

3. (15 pts.) Let $f(x) = \begin{cases} 2x & x < 1 \\ 5 & x = 1 \\ 4x - 2 & 1 < x < 3 \\ 7x & x \geq 3 \end{cases}$.

- (a) Find $\lim_{x \rightarrow 1^-} f(x)$.
(b) Find $\lim_{x \rightarrow 1^+} f(x)$.
(c) Does $\lim_{x \rightarrow 1} f(x)$ exist? If so, what is it?
(d) Is $f(x)$ continuous at $x = 1$? Why or why not?
(e) Is $f(x)$ differentiable at $x = 1$? Why or why not?

4. (25 pts.) Compute the first derivative of the following functions:

(a) $f(x) = x^5 - 4x^4 + 2x^3 - 5x^2 + 2x - 6$

(b) $g(t) = \cos(3t + 2) - 4 \sin(t^2 + 1)$

(c) $h(x) = (2x^3 + 5)^7(3x^2 + 7)^5$

(d) $k(x) = \frac{x^2 + 3x - 5}{\sqrt{x}}$

(e) $w(x) = \int_0^x \frac{t \, dt}{\sqrt{t^4 + 5t + 1}}$

5. (10 pts.) Use implicit differentiation to find the tangent line to the curve $4x^2 - 8xy + 3y^2 = 32$ at the point $(1, -2)$.

6. (15 pts.) An iron cylinder is being compressed in such a way that it maintains its volume. If the height of the cylinder is decreasing at 2 inches per minute, how fast is the radius increasing when the height is 10 inches and the radius is 4 inches?

7. (20 pts.) Sketch a graph of $y = f(x)$ where $f(x)$ satisfies ALL the following properties:
- (a) $f(-4) = 2$, $f(-2) = 3$, $f(0) = -1$
 - (b) $f(x)$ has vertical asymptotes at $x = -1$ and $x = 3$
 - (c) $\lim_{x \rightarrow -\infty} = 1$, $\lim_{x \rightarrow \infty} = -2$
 - (d) $f'(x) > 0$ on $(-\infty, -2) \cup (0, 3) \cup (3, \infty)$
 - (e) $f'(x) < 0$ on $(-2, -1) \cup (-1, 0)$
 - (f) $f''(x) > 0$ on $(-\infty, -4) \cup (-1, 3)$
 - (g) $f''(x) < 0$ on $(-4, -1) \cup (3, \infty)$

8. (15 pts.) Let $f(x) = x + \frac{9}{x}$.
- (a) On what intervals is $f(x)$ increasing?
 - (b) On what intervals is $f(x)$ concave up?
 - (c) Where does the graph of $f(x)$ have vertical asymptotes?

9. (15 pts.) A rectangular box is to have a square base and no top. If the base and sides of the box are to have a total surface area of 32 square inches, what are the dimensions of the box of maximum volume that can be obtained?

10. (10 pts.) Use Riemann sums with $n = 4$ rectangles and left-hand endpoints to estimate $\int_1^3 \frac{1}{x} dx$. Is this an overestimate or an underestimate?

11. (10 pts.) Find the exact area under the curve $y = \sin(3x)$ and above the x -axis from $x = 0$ to $x = \pi/4$.

12. (10 pts.) The acceleration of a particle is given by $a(t) = 8 \sin(2t) - 9 \cos(3t)$. If the initial position and velocity are given by $x(0) = 0$ and $v(0) = 0$, find the position $x(t)$.

13. (30 pts.) Evaluate the following integrals:
- (a) $\int \frac{x^3 + 3x + 1}{\sqrt{x}} dx$
 - (b) $\int_0^{\pi/2} \sec^2\left(\frac{x}{2}\right) dx$
 - (c) $\int_0^1 \frac{x^2}{(2x^3 + 1)^{3/2}} dx$
 - (d) $\int_0^1 5 - 2x + x^4 dx$