

MATH 229 LECTURE NOTES:

Higher Order Derivatives

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1. THE SECOND DERIVATIVE

Since we **love** differentiating so much, it seems natural to keep doing it.

That is, once we find the derivative of a function f , we may wish to find the derivative of the derivative f'

For example, if

$$f(x) = 7x^3 + 3x^2 + 11x - 9$$

then

$$f'(x) = 21x^2 + 6x + 11$$

The derivative of f' is

$$(f')'(x) = 42x + 6$$

- The derivative of the derivative is called the **second** derivative.
- What else would it be called?
- and is written $f''(x)$

2. HIGHER ORDER DERIVATIVES

- There is no need to stop at two.
- The derivative of the second derivative is called the **third** derivative (What else would it be called?)
- and is written $f'''(x)$
- continuing with derivatives of order 4, 5, 6, etcetera
- For $n > 3$ we write the n th derivative as
- $f^{(n)}(x)$
- Note the parentheses around the n

3. AN EXAMPLE

- For $f(x) = x^5$

- $f'(x) = 5x^4$
- $f''(x) = 5 \cdot 4x^3$
- $f'''(x) = 5 \cdot 4 \cdot 3x^2$
- $f^{(4)}(x) = 5 \cdot 4 \cdot 3 \cdot 2x$
- $f^{(5)}(x) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ is written $5!$
- and is called “5-factorial”
- For $n > 5$
- $f^{(n)}(x) = 0$

4. A HARDER EXAMPLE

- For $f(x) = \sqrt{1-x} = (1-x)^{\frac{1}{2}}$
- $f'(x) = \frac{1}{2}(1-x)^{-\frac{1}{2}} \cdot (-1) = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$
- $f''(x) = -\frac{1}{2} \cdot \frac{-1}{2}(1-x)^{-\frac{3}{2}} \cdot (-1) = -\frac{1}{2} \cdot \frac{1}{2}(1-x)^{-\frac{3}{2}}$
- $f'''(x) = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{-3}{2}(1-x)^{-\frac{5}{2}} \cdot (-1) = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2}(1-x)^{-\frac{5}{2}}$
- $f^{(4)}(x) = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{-5}{2}(1-x)^{-\frac{7}{2}} \cdot (-1) = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2}(1-x)^{-\frac{7}{2}}$
- We could keep going
- Notice that the each derivative is always negative
- The higher order derivatives will never equal zero

5. A TRIG EXAMPLE

For $f(x) = \sin(x)$ find

- $f'(x)$
- $f''(x)$
- $f^{(101)}(x)$

What's the pattern?

What happens if we change the function to

$$f(x) = \sin(7x)?$$

6. LEIBNITZ NOTATION

The second derivative in Leibnitz's notation is $\frac{d}{dx}\left(\frac{d}{dx}y\right)$ which is written as $\frac{d^2}{dx^2}y$ or $\frac{d^2y}{dx^2}$

In general

number	derivative
first	$\frac{dy}{dx}$
second	$\frac{d^2y}{dx^2}$
third	$\frac{d^3y}{dx^3}$
fourth	$\frac{d^4y}{dx^4}$
n -th	$\frac{d^ny}{dx^n}$

7. VELOCITY AND ACCELERATION

The rate at which the position of an automobile (or other vehicle) is changing with time is called its **velocity**.

Velocity is measured in miles per hour.

The rate at which the velocity changes is called the **acceleration**.

Acceleration is measured in miles per hour per hour.

8. FREE FALL

On Earth, when an object is dropped, the distance it falls after t seconds, if we ignore air resistance, is

$$s = 16t^2$$

Find the velocity and acceleration

$$v = \frac{ds}{dt} = 32t$$

$$a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = 32$$

called the acceleration due to gravity.

9. SKYDIVING

The distance a skydiver falls in 10 seconds is

$$s = 16 \cdot 10^2 = 1600$$

The skydiver's speed after 10 seconds is

$$v = 32 \cdot 10 = 320 \frac{\text{ft}}{\text{sec}} \times \frac{60 \text{ sec}}{\text{min}} = 19200 \frac{\text{ft}}{\text{min}}$$

To convert this to miles per hour it is convenient to know that 1 mile per hour equals 88 feet per minute:

$$v = 19200 \frac{\text{ft}}{\text{min}} \frac{1 \text{ mph}}{88 \text{ ft per min}} = 218.2 \text{ mph}$$

Is it reasonable to ignore air resistance?