

Math 230 Practice

Integral Worksheet Hints for Selected Problems

Do not read these until you have given each problem a fair try. These hints will get you started. In several cases they result in an integral which can easily be solved by standard techniques, such as integration by parts. For many of the integrals on the worksheet, there are several ways to approach them.

1. Substitute $u = 1 + e^x$.
2. Substitute $u = e^x$ (so that $e^{2x} = u^2$).
3. Substitute $x = t^2$.
4. Write $\frac{1}{1 + e^{2x}} = \frac{1 + e^{2x}}{1 + e^{2x}} - \frac{e^{2x}}{1 + e^{2x}}$.
5. Substitute $x = -t^2$.
6. Substitute $t = \ln x$ (so that $x = e^t$).
7. Use the trig identity $\sin 2x = 2 \sin x \cos x$ and substitute $u = 1 + \sin^2 x$.
11. By parts: $u(\ln x)^2$, $dv = dx$ reduces to an integral involving $\ln x$ to the first power.
12. Substitute $u = 1 + \ln x$.
13. Substitute $t = x^2$.
15. Convert to sin and cos.
17. Convert to sec and tan:

$$\frac{1}{1 - \sin x} = \frac{1}{1 - \sin x} \frac{1 + \sin x}{1 + \sin x} = \frac{1 + \sin x}{1 - \sin^2 x} = \frac{1 + \sin x}{\cos^2 x} = \dots$$

18. Step 1: Substitute $t = e^x$. Step 2: By parts, $u = \tan^{-1} t$, $dv = \frac{1}{t^2} dt$. Step 3: Integrate $\int \frac{1}{t(1+t^2)}$ by partial fractions.

$$20. \frac{\sin^2 x - \cos^2 x}{\cos x} = \frac{1 - 2\cos^2 x}{\cos x} = \sec x - 2\cos x.$$

21-23. Substitute $x = \sqrt{3} \sin t$.

24. Write $(x^2 - 4x + 4)(x^2 - 4x + 5) = (x - 2)^2((x - 2)^2 + 1)$. Then substitute $t = x - 2$.

26. Convert to sec and tan:

$$\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} \frac{1/\cos^2 x}{1/\cos^2 x} = \frac{\sec^2 x}{a^2 \tan^2 x + b^2}.$$

Now substitute $u = \tan x$.

27. Multiply the integrand by $\frac{1 - (\cos x + \sin x)}{1 - (\cos x + \sin x)}$ and use the fact that $\sin^2 x + \cos^2 x = 1$ after you expand the denominator.

$$30. \text{ Start with the integral } I = \int \frac{5x^5 + x}{(x^5 + x + 1)^2} dx = \int x \frac{5x^4 + 1}{(x^5 + x + 1)^2} dx.$$

(i) Apply integration by parts with $u = x$ and $dv = \frac{5x^4 + 1}{(x^5 + x + 1)^2} dx$.

(ii) Now write the numerator of the integral I as $5x^5 + x = (4x^5 - 1) + (x^5 + x + 1)$ and break I into the sum of two integrals.

31. Substitute $x = t^2$.

32. First use addition and subtraction formulas for $\cos(x + 2x)$ and $\cos(x - 2x)$ to write $\sin x \sin 2x$ in terms of $\cos(-x)$ (which equals $\cos x$) and $\cos 3x$. Now you have to do this process again, including the factor $\sin 3x$.