

INVERSE TRIG FUNCTIONS

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1. INVERSE TRIG FUNCTIONS

The number $\theta = \sin^{-1}(x)$ is defined to be the angle whose Sine is x , that is $\sin(\theta) = x$.

There is a problem, there are infinitely many angles whose sine is the same, because the sine curve goes between -1 and 1 infinitely many times.

Which angle do we use for $\sin^{-1}(x)$?

The solution is to restrict the sine curve so that it is one-to-one.

2. RESTRICT SIN, COS, TAN

function	domain	range
$y = \sin x$	$-\pi/2 \leq x \leq \pi/2$	$-1 \leq y \leq 1$
$y = \cos x$	$0 \leq x \leq \pi$	$-1 \leq y \leq 1$
$y = \tan x$	$-\pi/2 < x < \pi/2$	all reals

3. RESTRICT SEC, CSC, COT

function	domain	range
$y = \sec x$	$[0, \frac{\pi}{2}) \cup [\pi, 3\pi/2)$	$ y \geq 1$
$y = \csc x$	$(0, \frac{\pi}{2}] \cup (\pi, 3\pi/2]$	$ y \geq 1$
$y = \cot x$	$0 < x < \pi$	all reals

4. INVERSE TRIG DEFINITIONS

function	domain	range
$y = \sin x$	$-\pi/2 \leq x \leq \pi/2$	$-1 \leq y \leq 1$
$y = \cos x$	$0 \leq x \leq \pi$	$-1 \leq y \leq 1$
$y = \tan x$	$-\pi/2 < x < \pi/2$	all reals

function	domain	range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\pi/2 \leq y \leq \pi/2$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	all reals	$-\pi/2 < y < \pi/2$

5. INVERSE TRIG DEFINITIONS

function	domain	range
$y = \sec x$	$[0, \frac{\pi}{2}) \cup [\pi, 3\pi/2)$	$ y \geq 1$
$y = \csc x$	$(0, \frac{\pi}{2}] \cup (\pi, 3\pi/2]$	$ y \geq 1$
$y = \cot x$	$0 < x < \pi$	all reals

function	domain	range
$y = \sec^{-1} x$	$ x \geq 1$	$[0, \frac{\pi}{2}) \cup [\pi, 3\pi/2)$
$y = \csc^{-1} x$	$ x \geq 1$	$(0, \frac{\pi}{2}] \cup (\pi, 3\pi/2]$
$y = \cot^{-1} x$	all reals	$0 < y < \pi$

6. EVALUATIONS

Find $\sin^{-1}(-\frac{1}{2})$:

Find $\tan^{-1}(\sqrt{3})$:

Find $\sec^{-1}(-\sqrt{2})$:

7. EQUATIONS

Solve for x :

$$\tan^{-1}(2x - 3) = \frac{\pi}{4}$$

8. CALCULATIONS

Given $\theta = \sec^{-1}(\sqrt{5}/2)$,

Find $\sin(\theta)$

Find $\tan(\theta)$

9. DERIVATIVES

Find $\frac{d}{dx} \sin^{-1}(x)$

Find $\frac{d}{dx} \tan^{-1}(x)$

Find $\frac{d}{dx} \sec^{-1}(x)$

10. INVERSE TRIG DERIVATIVES

function	derivative
$y = \sin^{-1} x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \tan^{-1} x$	$y' = \frac{1}{1+x^2}$
$y = \sec^{-1} x$	$y' = \frac{1}{x\sqrt{x^2-1}}$

Cofunction rule: Replace each trig function with its cofunction and place a minus sign on the right hand side.