

1. MATH 353

Isosceles Triangle “Proof”

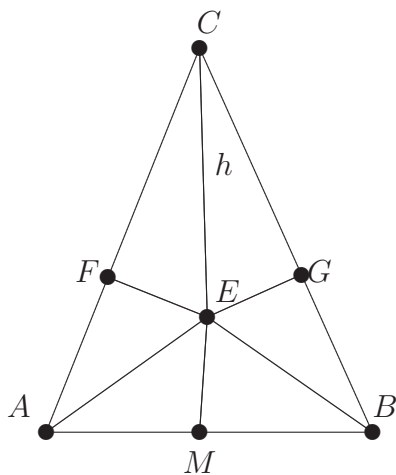
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2. ISOSCELES TRIANGLE PARADOX



ray h = bisector of $\angle C$

M = midpoint of $\overline{AB}$

$\perp$ bisector of $\overline{AB}$ meets h at E

Drop perp from E to $\overline{AC}$ at F

Drop perp from E to $\overline{BC}$ at G

Draw segments $\overline{AE}$ and $\overline{BE}$

3. STEP 1. $\triangle CFE \cong \triangle CGE$

$\triangle CFE \cong \triangle CGE$ by Angle–Angle–Side.

Each is a right triangle with $\overline{CE}$ as a hypotenuse and

$\angle FCE = \angle GCE$ since $\overrightarrow{CE}$ bisects $\angle C$.

4. STEP 2. SEGMENT EQUALITIES

$$EF = EG \text{ and } CF = CG$$

[congruent parts of the congruent triangles $\triangle CFE$ and $\triangle CGE$].

5. STEP 3. $\triangle AEM \cong \triangle BEM$

$\triangle AEM \cong \triangle BEM$ by Side–Angle–Side.

Both are right triangles,

$AM = BM$ since M is the midpoint of $\overline{AB}$,

and they share side $\overline{EM}$.

6. STEP 4. $AE = BE$

CPCT

7. STEP 5. $\triangle AEF \cong \triangle BEG$

By Hypotenuse–Leg.

Both are right triangles,

$EF = EG$ from step 2, and

$AE = BE$ from step 4.

8. STEP 6. $FA = GB$

CPCT

9. STEP 7. COUP DE GRAS

Combining steps 2 and 6, we have

$$CA = CF + FA = CG + GB = CB$$

10. CONCLUSION

Since $CA = CB$ it follows that

Theorem. All triangles are isosceles.

11. YIKES

Something is very wrong here!

We have just “proved” a statement that is categorically false.

Clearly not all triangles are isosceles.

Challenge: find the flaw in the proof.