

1. HOW TO FIND LIMITS ALGEBRAICALLY

Professor Richard Blecksmith

rblecksm@niu.edu

Dept. of Mathematical Sciences

Northern Illinois University

<http://math.niu.edu/~richard/Math229>

2. THE PLUG IN RULE

To compute $\lim_{x \rightarrow a} f(x)$, plug in $x = a$ into $f(x)$,

If $f(a)$ exists, then 99 times out of 100, it is the limit.

Let $f(x) = \frac{x^2-1}{x^3-1}$.

To find $\lim_{x \rightarrow 2} f(x)$, plug in $x = 2$.

$$\lim_{x \rightarrow 2} \frac{x^2 - 1}{x^3 - 1}$$

$$= \frac{2^2-1}{2^3-1}$$

$$= \frac{4-1}{8-1} = \frac{3}{7} \quad [\text{My final answer.}]$$

3. PLUG IN RULE FAILS

If $f(a)$ does not exist, then we need to work harder.

If $f(x) = \frac{x^2-1}{x^3-1}$, then plugging in $x = 1$ gives

You **CANNOT** write: $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1} = \frac{0}{0}$.

What now?

One way is to plug in a value of x close to 1.

Take $x = 1.001$. My calculator says $1.001^2 - 1 = .002001$ and $1.001^3 - 1 = .003003001$.

So $f(1.001) = \frac{.002001}{.003003001} = .66633344444440$, which is approximately $2/3$.

4. FACTOR METHOD

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1} \\ &= \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{(x - 1)(x^2 + x + 1)} \end{aligned}$$

[How do you know how to factor $x^3 - 1$?]

$$\begin{aligned} &= \lim_{x \rightarrow 1} \frac{x + 1}{x^2 + x + 1} \quad [\text{Cancel } x - 1] \\ &= \frac{1+1}{1^2+1+1} \quad [\text{Plug in } x = 1.] \\ &= \frac{2}{3} \quad [\text{My final answer.}] \end{aligned}$$

5. COMBINE FRACTIONS METHOD

$$\begin{aligned} & \lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{\frac{2}{2x} - \frac{x}{2x}}{x - 2} \quad [\text{Common denominator is } 2x] \\ &= \lim_{x \rightarrow 2} \frac{\frac{2-x}{2x}}{x - 2} \quad [\text{Subtract the two fractions}] \\ &= \lim_{x \rightarrow 2} \frac{2-x}{2x} \frac{1}{x-2} \quad [\text{Invert and multiply}] \end{aligned}$$

6. COMBINE FRACTIONS METHOD CONT'D

$$= \lim_{x \rightarrow 2} \frac{2-x}{2x} \frac{1}{x-2}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 2} \frac{-(x-2)}{2x} \frac{1}{x-2} \quad [\text{since } 2-x = -(x-2)] \\
&= \lim_{x \rightarrow 2} \frac{-1}{2x} \quad [\text{Cancel } x-2] \\
&= \frac{-1}{2 \cdot 2} \quad [\text{Plugging in } x = 2] \\
&= -\frac{1}{4} \quad [\text{My final answer.}]
\end{aligned}$$

7. THE SQUARE ROOT TRICK

$$\begin{aligned}
&\lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3} \\
&= \lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3} \frac{\sqrt{x} + 3}{\sqrt{x} + 3} \quad [\text{This is the trick!}]
\end{aligned}$$

It's called rationalizing the denominator.]

$$= \lim_{x \rightarrow 9} \frac{(x^2 - 81)(\sqrt{x} + 3)}{x - 9}$$

$$[(A - B)(A + B) = A^2 - B^2 \text{ and } \sqrt{x}^2 = x]$$

[Use the factor method now]

8. THE SQUARE ROOT TRICK CONT'D

$$\begin{aligned}
&= \lim_{x \rightarrow 9} \frac{(x^2 - 81)(\sqrt{x} + 3)}{x - 9} \\
&= \lim_{x \rightarrow 9} \frac{(x - 9)(x + 9)(\sqrt{x} + 3)}{x - 9} \quad [\text{Factor } x^2 - 81] \\
&= \lim_{x \rightarrow 9} (x + 9)(\sqrt{x} + 3) \quad [\text{Cancel } x - 9] \\
&= (9 + 9)6 \quad [\text{Plug in } x = 9] \\
&= 108 \quad [\text{My final answer.}]
\end{aligned}$$