

Nim

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The Game of Nim

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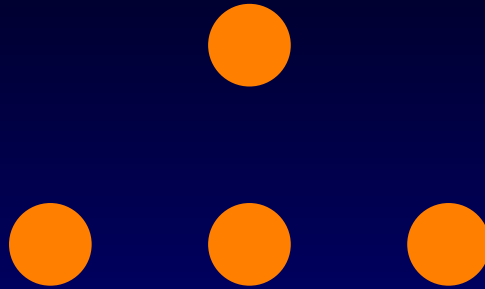
They must take at least one coin.

The player to take the last penny wins.

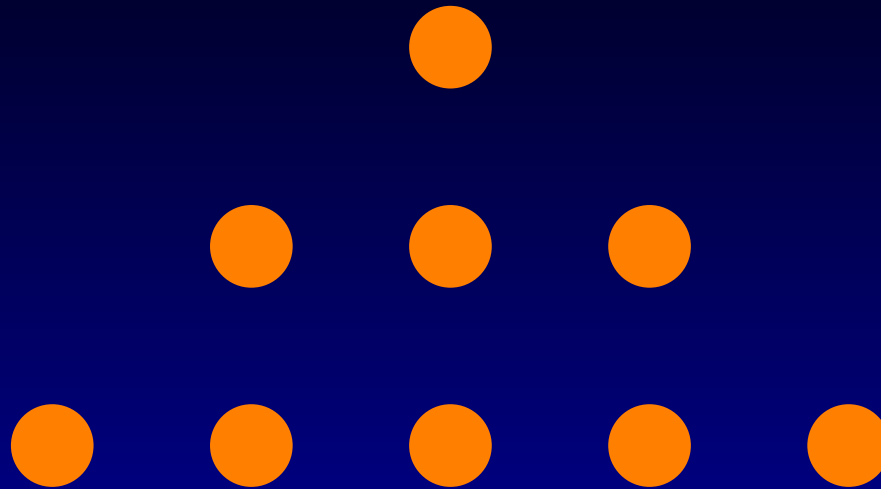
One Nim Configuration



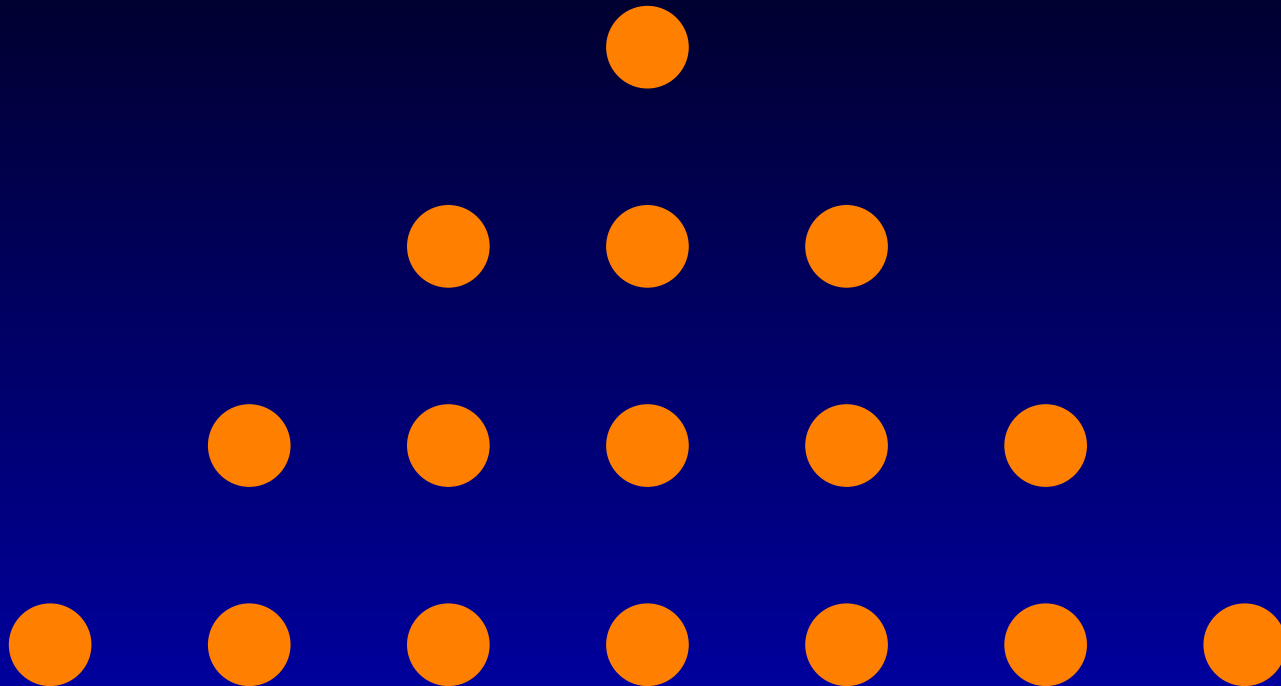
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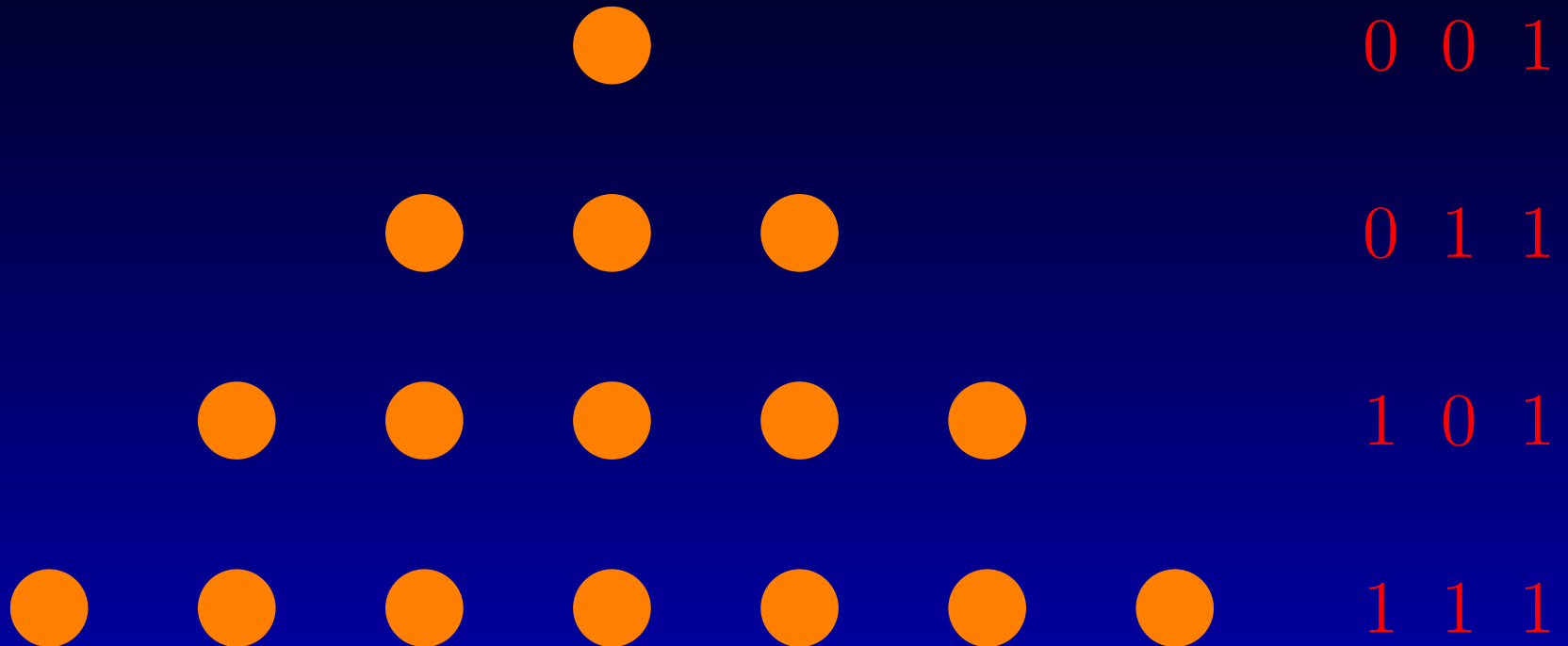
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If the sum of each of the binary columns is odd, the pennies are in a **balanced configuration**.

Removing one or more pennies from any one row will result in an unbalanced configuration, meaning that the sum of at least one column is odd.

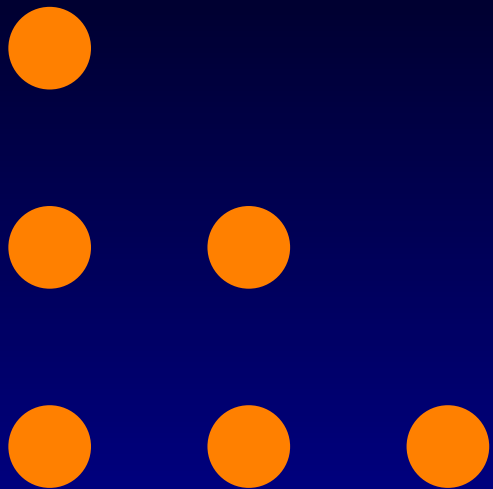
Another Nim Configuration



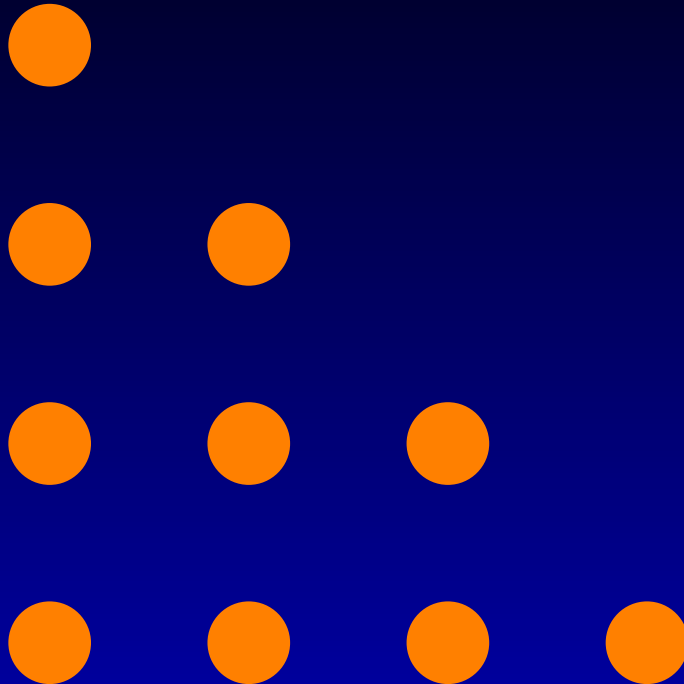
Another Nim Configuration



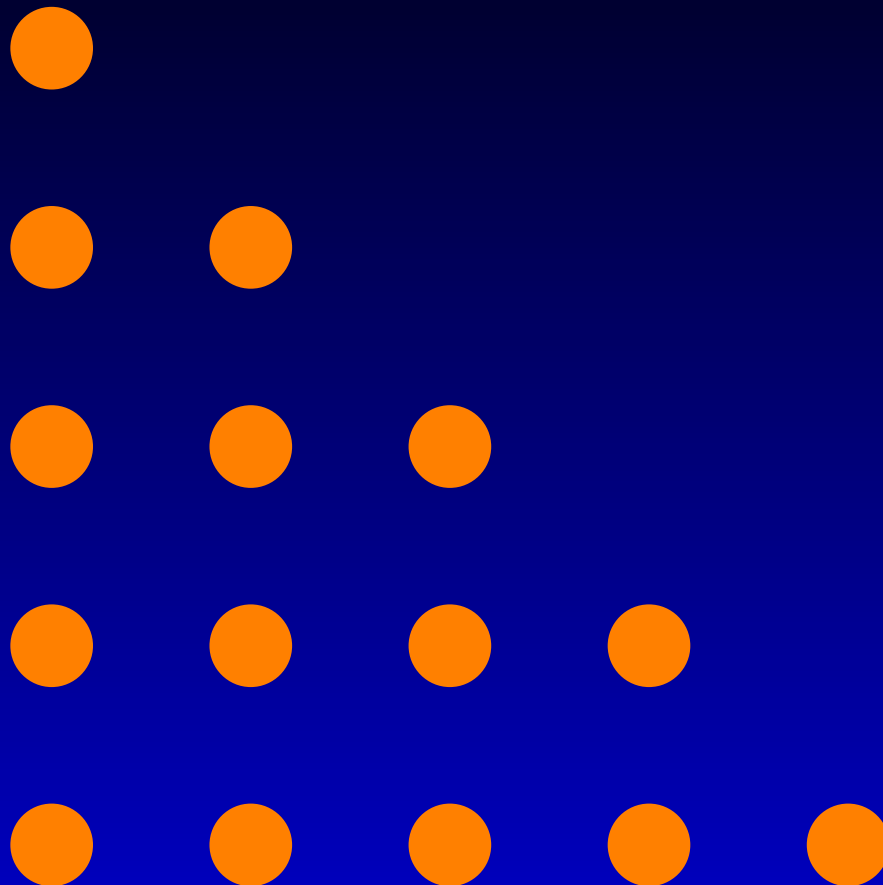
Another Nim Configuration



Another Nim Configuration



Another Nim Configuration



Another Nim Configuration

●					0 0 1
●	●				0 1 0
●	●	●			0 1 1
●	●	●	●		1 0 0
●	●	●	●	●	1 0 1

Another Nim Configuration

●						0	0	1
●	●					0	1	0
●	●	●				0	1	1
●	●	●	●			1	0	0
●	●	●	●	●		1	0	1

What move will balance the configuration?

Strategy for Winning Nim

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You can always find a move which will restore the configuration to the balanced state.