

INTEGRATION BY PARTS

Professor Richard Blecksmith

richard@math.niu.edu

Dept. of Mathematical Sciences

Northern Illinois University

<http://math.niu.edu/~richard/Math230>

1. THE IDEA

Use the product rule to differentiate $x e^x$:

$$\begin{aligned}\frac{d}{dx} x e^x &= \frac{d}{dx} x e^x + x \frac{d}{dx} e^x \\ &= 1 \cdot e^x + x e^x = e^x + x e^x\end{aligned}$$

$$\text{Integrate: } \int \frac{d}{dx} x e^x = \int e^x + \int x e^x$$

Which of these three integrals can we do? Answer: The first and the second.

$$\text{So } x e^x = e^x + \int x e^x$$

$$\text{Which implies: } \int x e^x = e^x - x e^x + C$$

2. INTEGRATION BY PARTS

Based on the product rule:

$$(uv)' = u \cdot v' + v \cdot u',$$

which can be rewritten as

$$u \cdot v' = (uv)' - v \cdot u'$$

$$\begin{aligned}\text{So } \int u \cdot v' &= \int (uv)' - \int v \cdot u' \\ &= (uv) - \int v \cdot u'\end{aligned}$$

3. EXAMPLE 1

Integrate $\int x e^x$.

What should we make u ?

What should we make v' ?

4. DIFFERENTIAL NOTATION

It is common to write the integration by parts equation as:

$$\text{So } \int u \cdot dv = uv - \int v \cdot du,$$

where $du = u' dx$ and $dv = v' dx$.

The trick is to figure out what to make u and dv .

5. EXAMPLE 2

Integrate $\int x^2 \ln x$.

Decision time: do we use

$$u = x^2 \text{ and } dv = \ln x dx$$

$$\text{or } u = \ln x \text{ and } dv = x^2 dx ?$$

6. EXAMPLE 3

Integrate $\int \sin^{-1} x dx$.

Trick: Make $dv = 1 \cdot dx$.

7. EXAMPLE 4

Integrate $\int x^2 \sin x \, dx$.

In this example, we will need to use integration by parts twice.

8. EXAMPLE 5

Integrate $\int e^x \cos x \, dx$.

Make $dv = e^x \, dx$.

9. EXAMPLE 6

Integrate $\int e^{\sqrt{x}} \, dx$.

Make substitution $t = \sqrt{x}$.

Be careful with the letters u and v . If we substitute $u = \sqrt{x}$, we run into problems when we want to integrate by parts, because the letter u is spoken for.

Mean test problem: Integrate $\int u \cdot e^u \, du$.