

Guess the Number

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Guess the next term:

1, 3, 5,

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1, 3, 5,

1, 4, 9,

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1, 3, 5,

1, 4, 9,

1, 3, 6,

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Two choices:

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(i) if we see the pattern as $1, 1 + 1 = 2, 2 + 2 = 4$, then the next number should be $4 + 3 = 7$.

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(i) if we see the pattern as $1, 1 + 1 = 2, 2 + 2 = 4$, then the next number should be $4 + 3 = 7$.

(ii) if we see the pattern as $1, 1 = 2^0, 2 = 2^1, 4 = 2^2$, then the next number should be $2^3 = 8$.

Regions in the x - y plane

Into how many regions do n random line separate the x - y plane?

no line: 1 region

Regions in the x - y plane

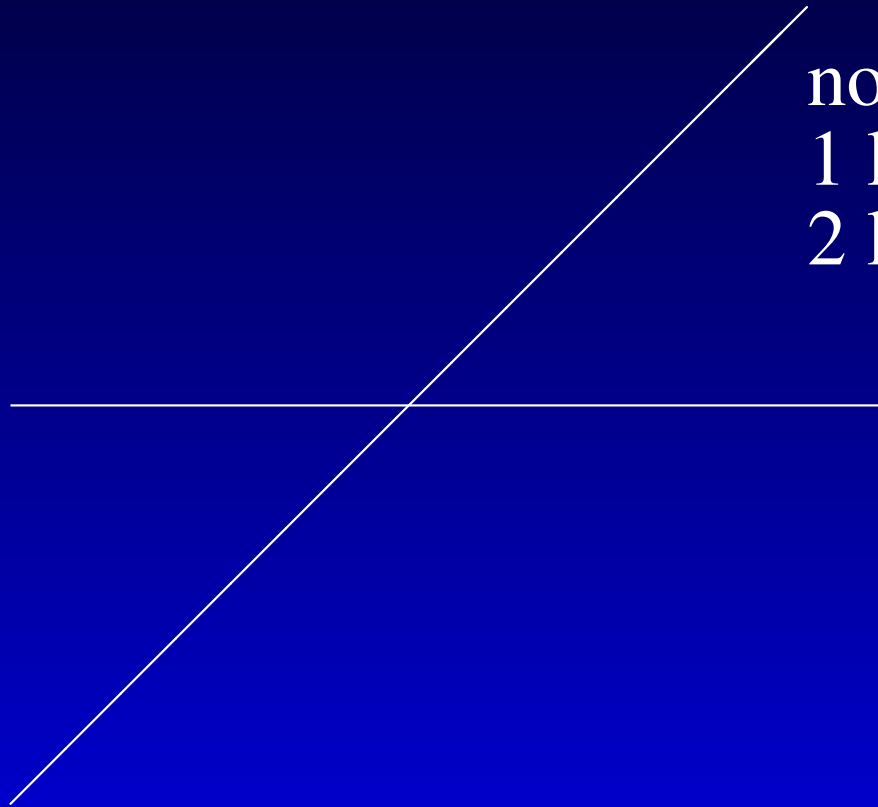
Into how many regions do n random line separate the x - y plane?

no line: 1 region
1 line: 2 regions



Regions in the x - y plane

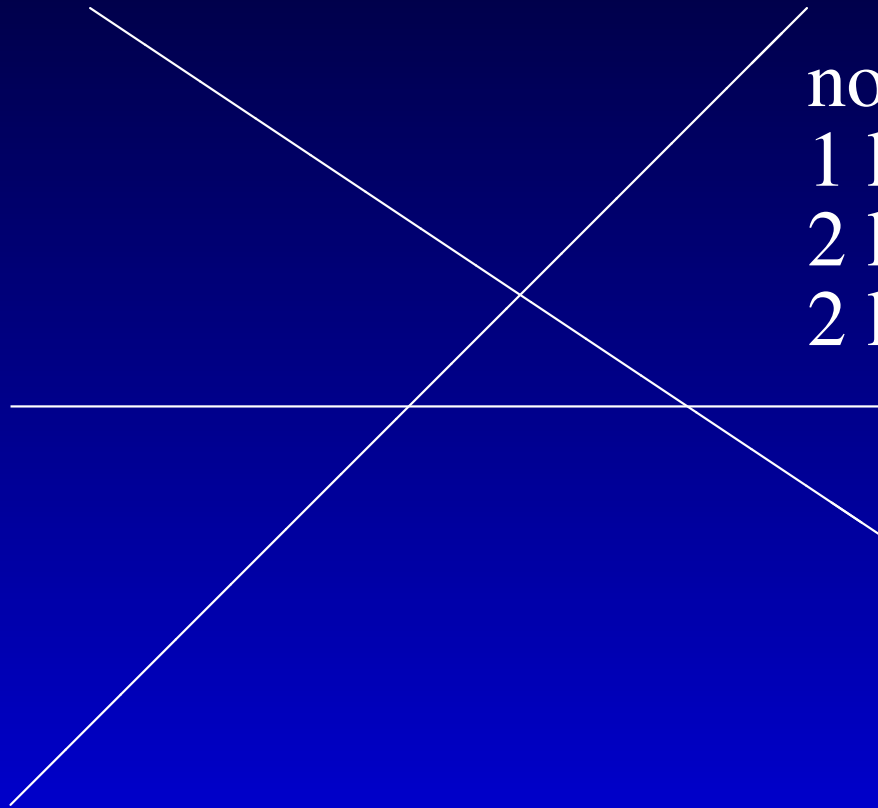
Into how many regions do n random line separate the x - y plane?



no line: 1 region
1 line: 2 regions
2 lines: 4 regions

Regions in the $x-y$ plane

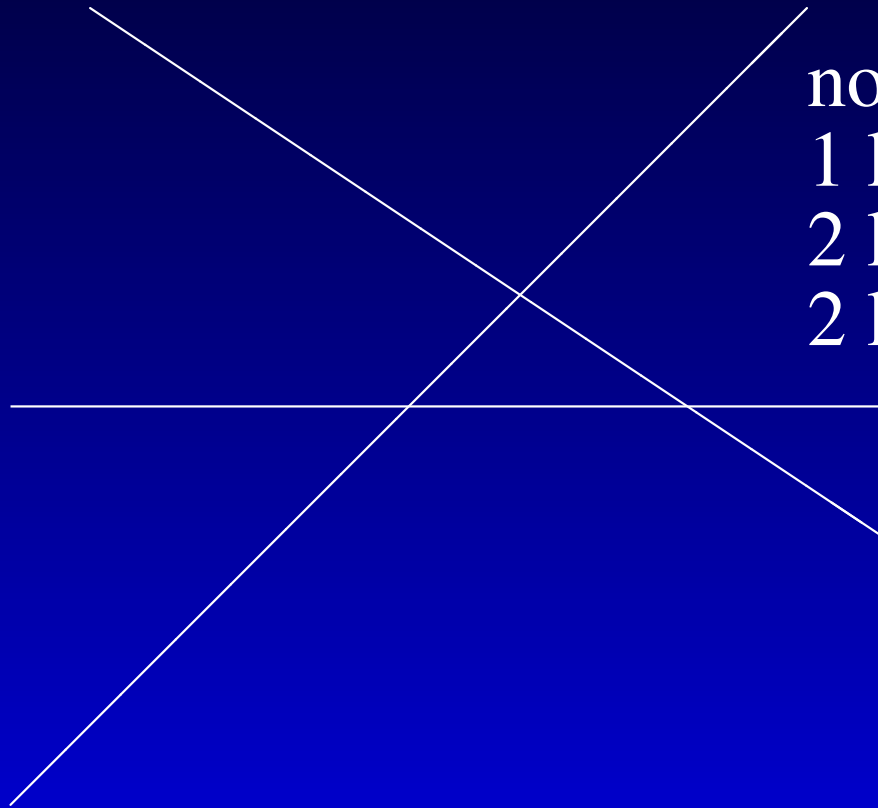
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Regions in the $x-y$ plane

Into how many regions do n random line separate the $x-y$ plane?



no line: 1 region
1 line: 2 regions
2 lines: 4 regions
3 lines: 7 regions

Flipping Coins

The number of ways to flip a coin is 2:

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heads (H) or tails (T).

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HHH THH

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The numbers 2, 4, 8 double.

More guessing

2, 6, 12,

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2, 6, 12,

2, 3, 5,

More guessing

2, 6, 12,

2, 3, 5,

2, 3, 5, 7, 11, 13,

More guessing

2, 6, 12,

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These are the **Primes Numbers**

Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13,

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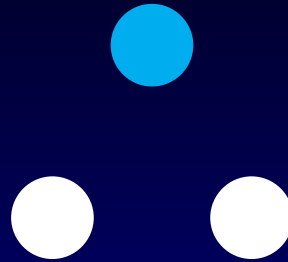
For other sequences you can give an **recursive formula** for the n -th term a_n in terms of the values of previous terms.

Triangular Numbers



$$T_1 = 1$$

Triangular Numbers



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Triangular Numbers



$$T_1 = 1$$



$$T_2 = 3$$

Triangular Numbers



$$T_1 = 1$$



$$T_2 = 3$$



Triangular Numbers



$$T_1 = 1$$



$$T_2 = 3$$



$$T_3 = 6$$

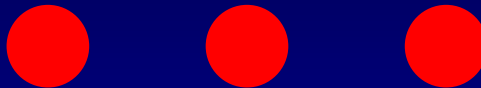
Triangular Numbers



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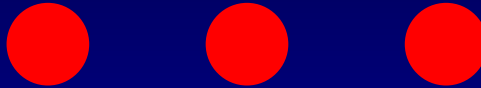
Triangular Numbers



$$T_1 = 1$$



$$T_2 = 3$$



$$T_3 = 6$$



$$T_4 = 10$$

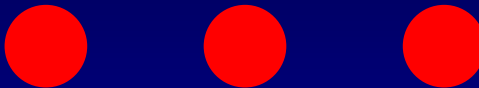
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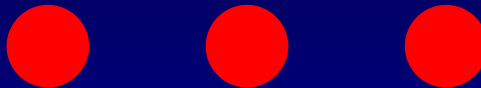
Triangular Numbers



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$$T_5 = 15$$

Exploring Triangular Numbers

The sequence of triangular numbers T_n is

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Notice that

$$T_n = 1 + 2 + 3 + \dots + n,$$

the sum of the numbers from 1 to n .

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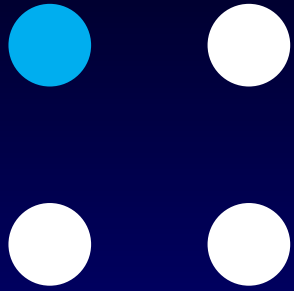
What is T_{100}

Square Numbers



$$S_1 = 1$$

Square Numbers



$$S_1 = 1$$

Square Numbers



$$S_1 = 1$$



$$S_2 = 4$$

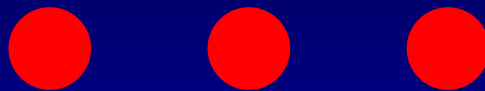
Square Numbers



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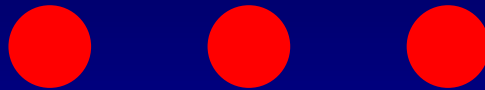
Square Numbers



$$S_1 = 1$$



$$S_2 = 4$$



$$S_3 = 9$$

Square Numbers



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$$S_2 = 4$$



$$S_3 = 9$$



Square Numbers



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$$S_2 = 4$$

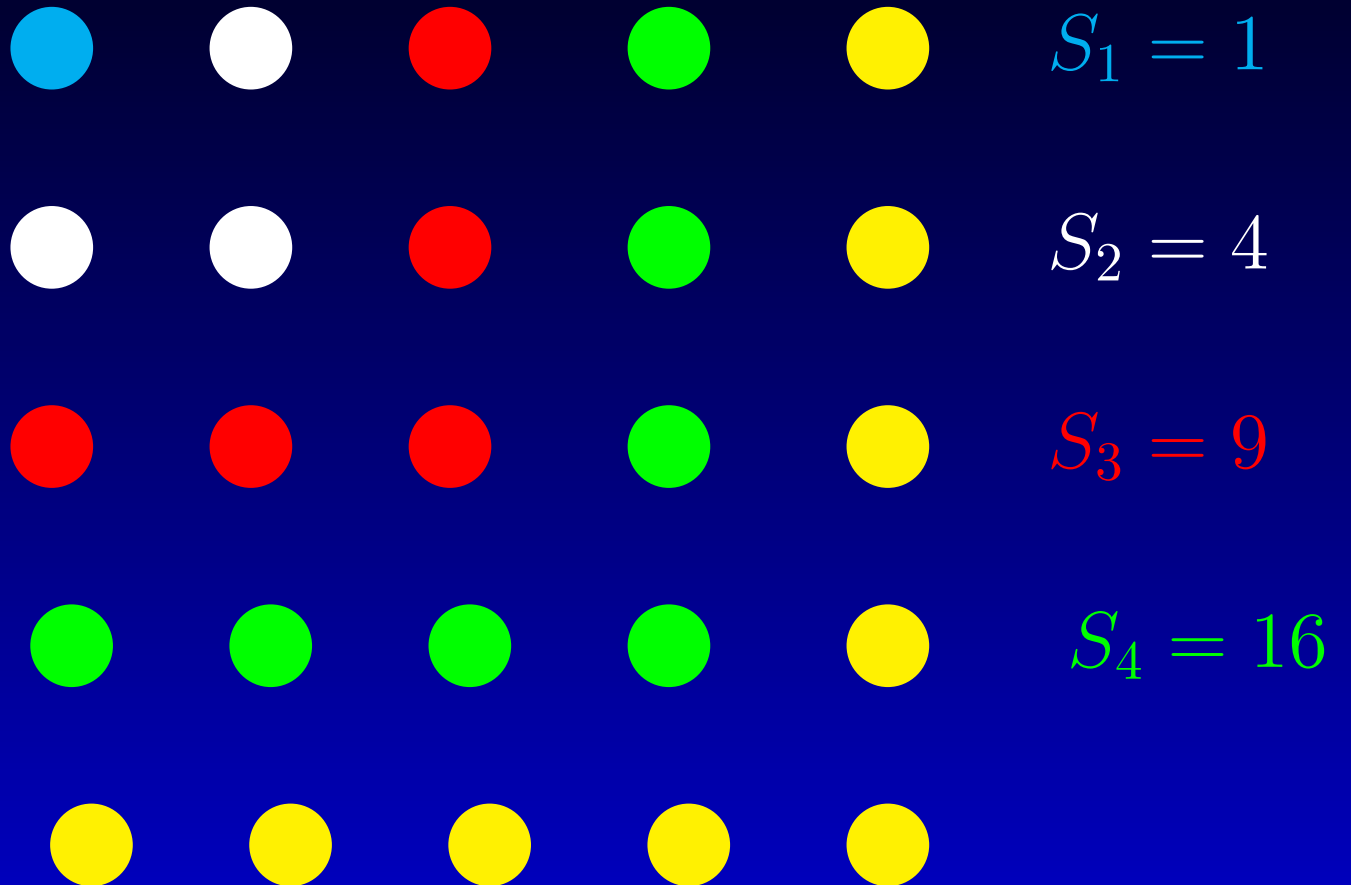


$$S_3 = 9$$

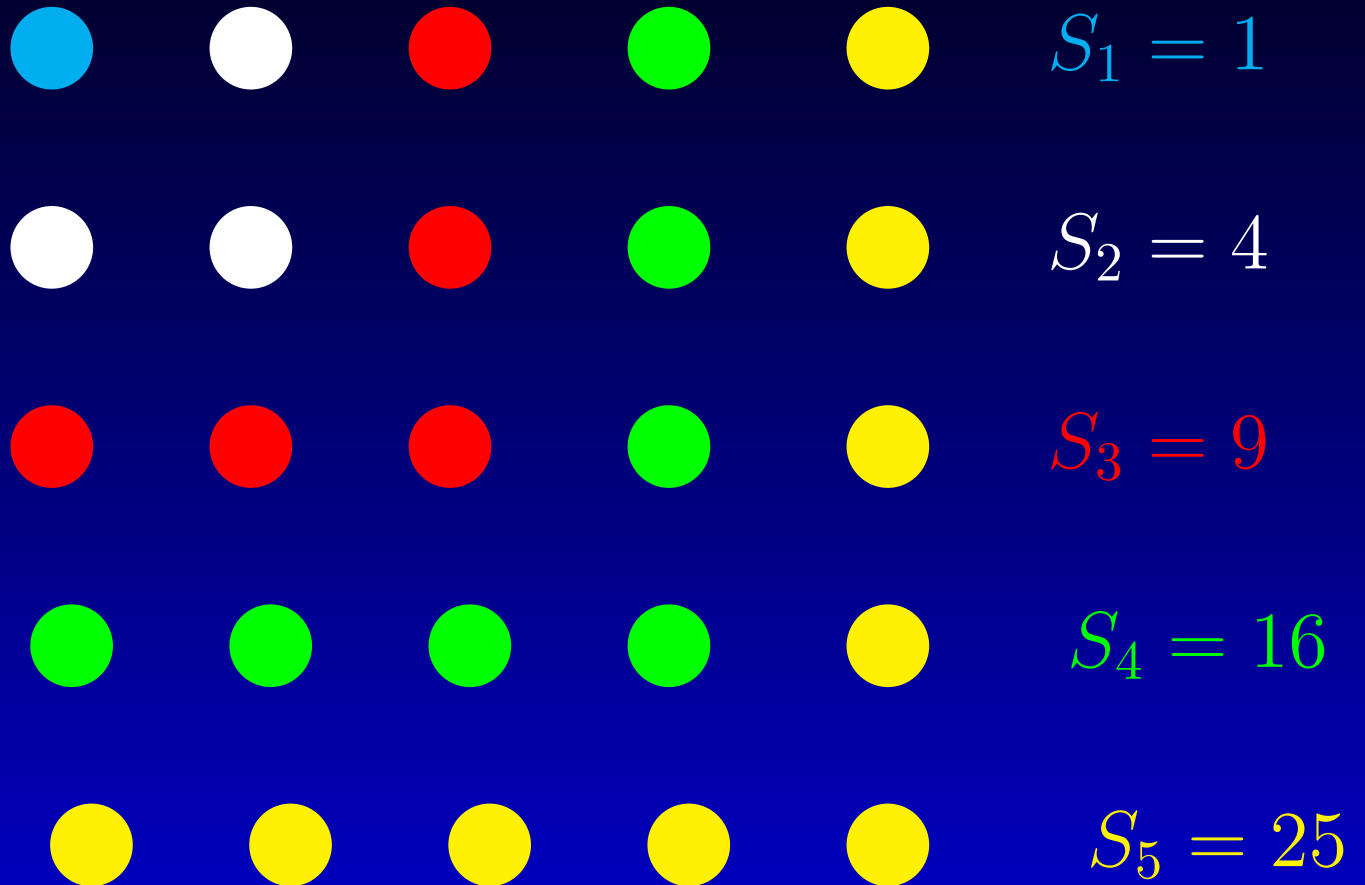


$$S_4 = 16$$

Square Numbers



Square Numbers



Exploring Square Numbers

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1, 4, 9, 16, 25, 36, ...

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Hint: Look at the diagram.

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Hint: Look at the diagram.

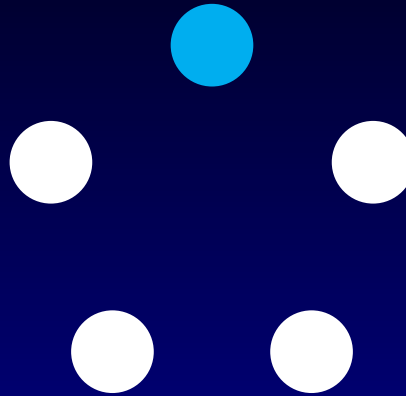
Can you write S_5 as a sum of 5 integers?

Pentagonal Numbers



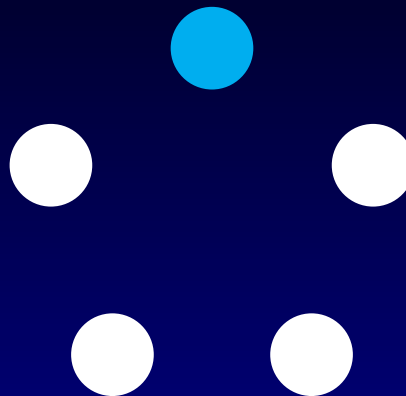
$$P_1 = 1$$

Pentagonal Numbers



$$P_1 = 1$$

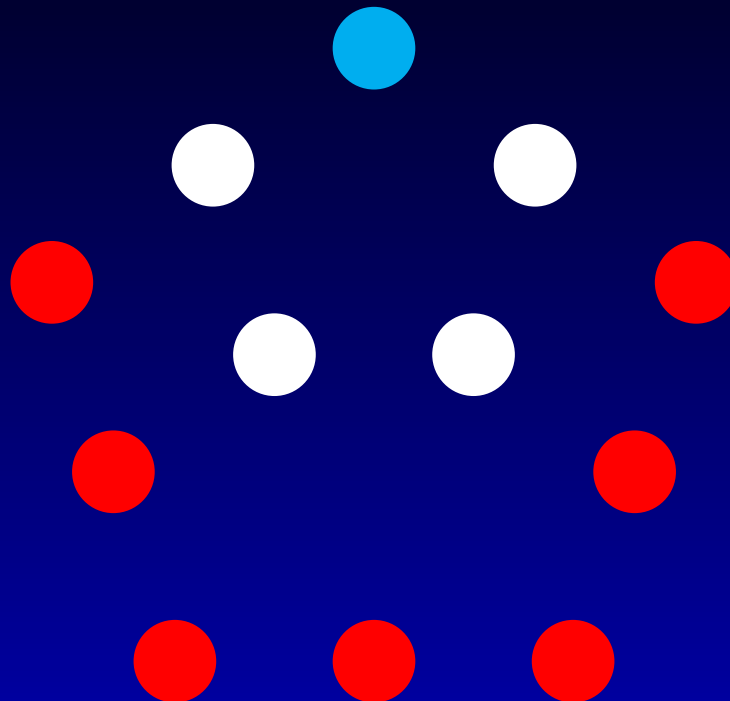
Pentagonal Numbers



$$P_1 = 1$$

$$P_2 = 5$$

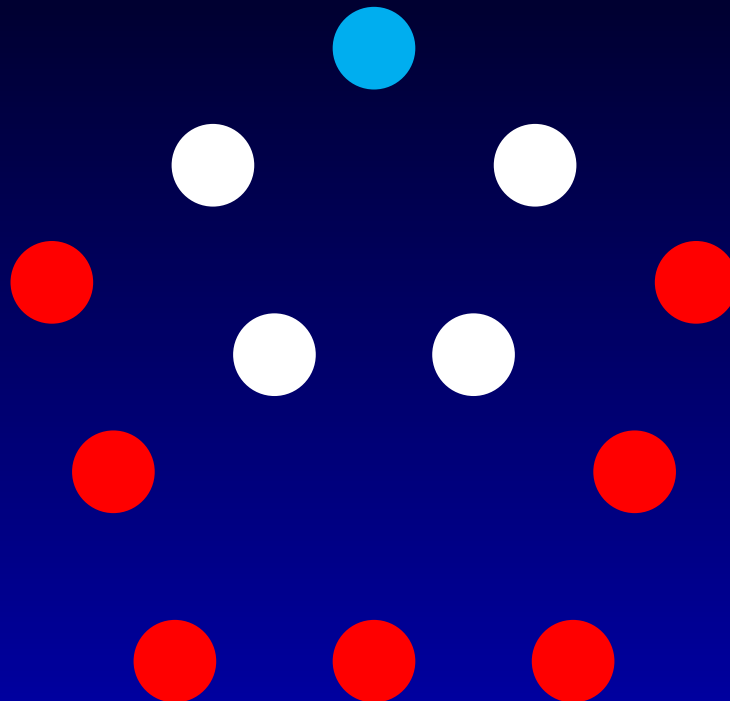
Pentagonal Numbers



$$P_1 = 1$$

$$P_2 = 5$$

Pentagonal Numbers

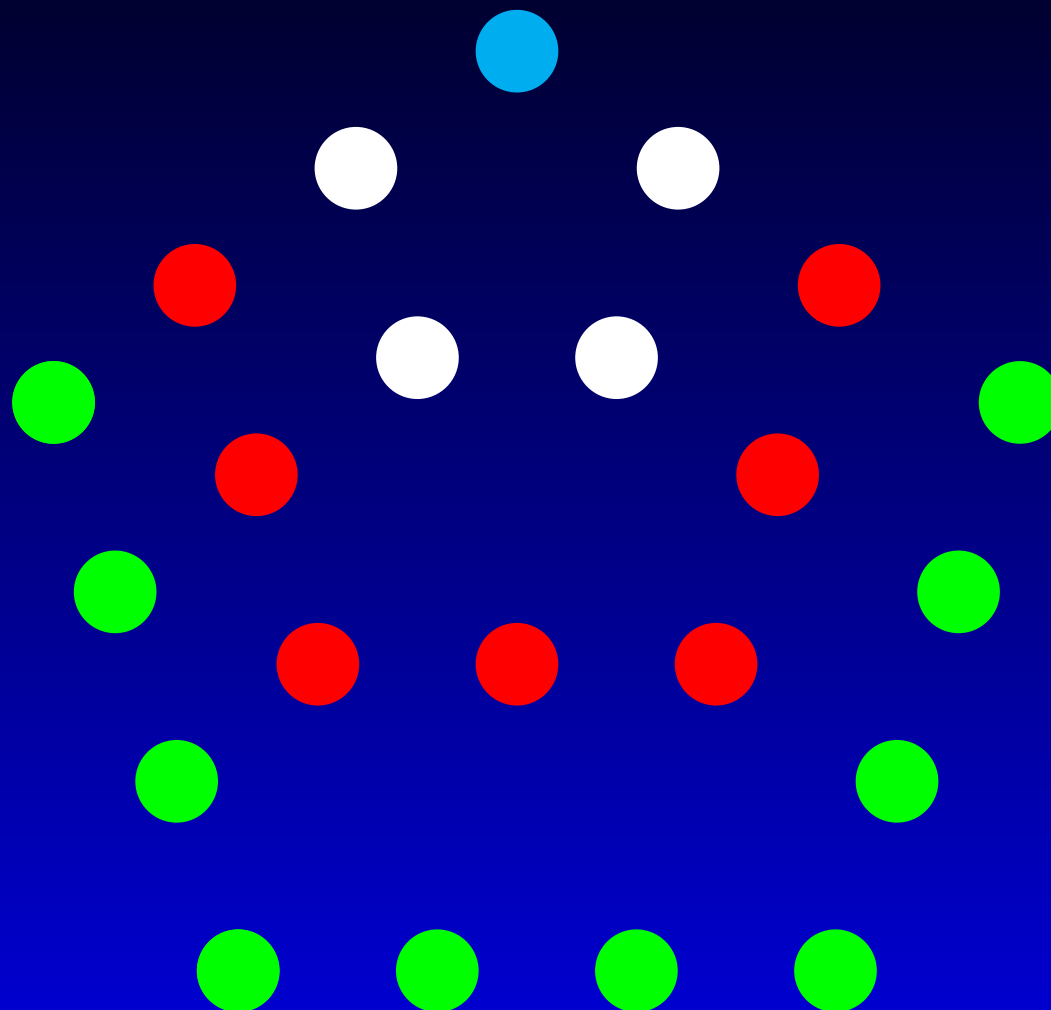


$$P_1 = 1$$

$$P_2 = 5$$

$$P_3 = 12$$

Pentagonal Numbers

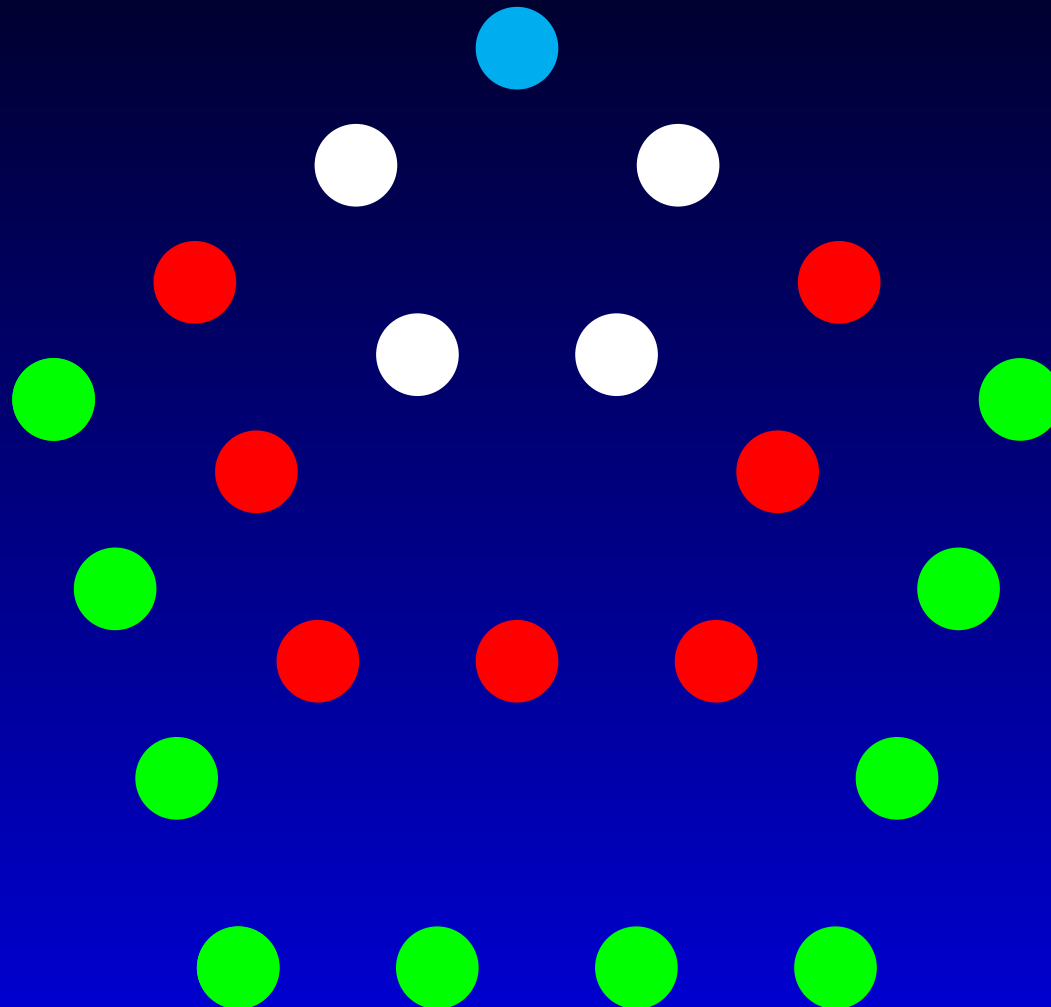


$$P_1 = 1$$

$$P_2 = 5$$

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Pentagonal Numbers



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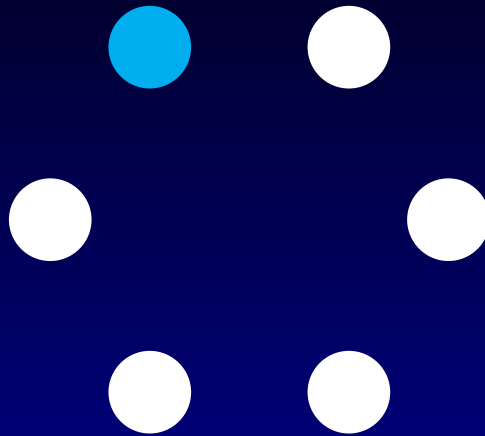
$$P_4 = 22$$

Hexagonal Numbers



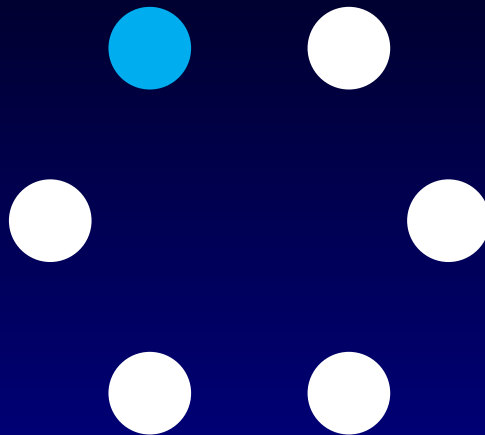
$$H_1 = 1$$

Hexagonal Numbers



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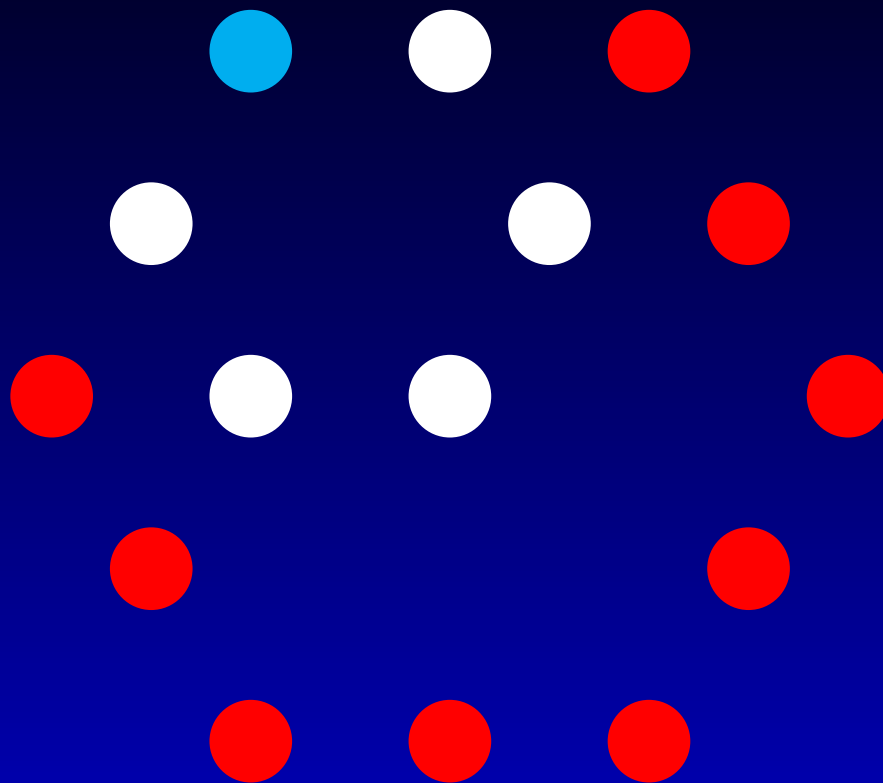
Hexagonal Numbers



$$H_1 = 1$$

$$H_2 = 6$$

Hexagonal Numbers



$$H_1 = 1$$

$$H_2 = 6$$

Hexagonal Numbers



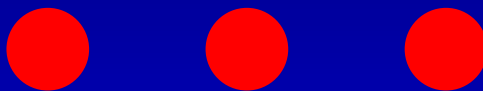
$$H_1 = 1$$



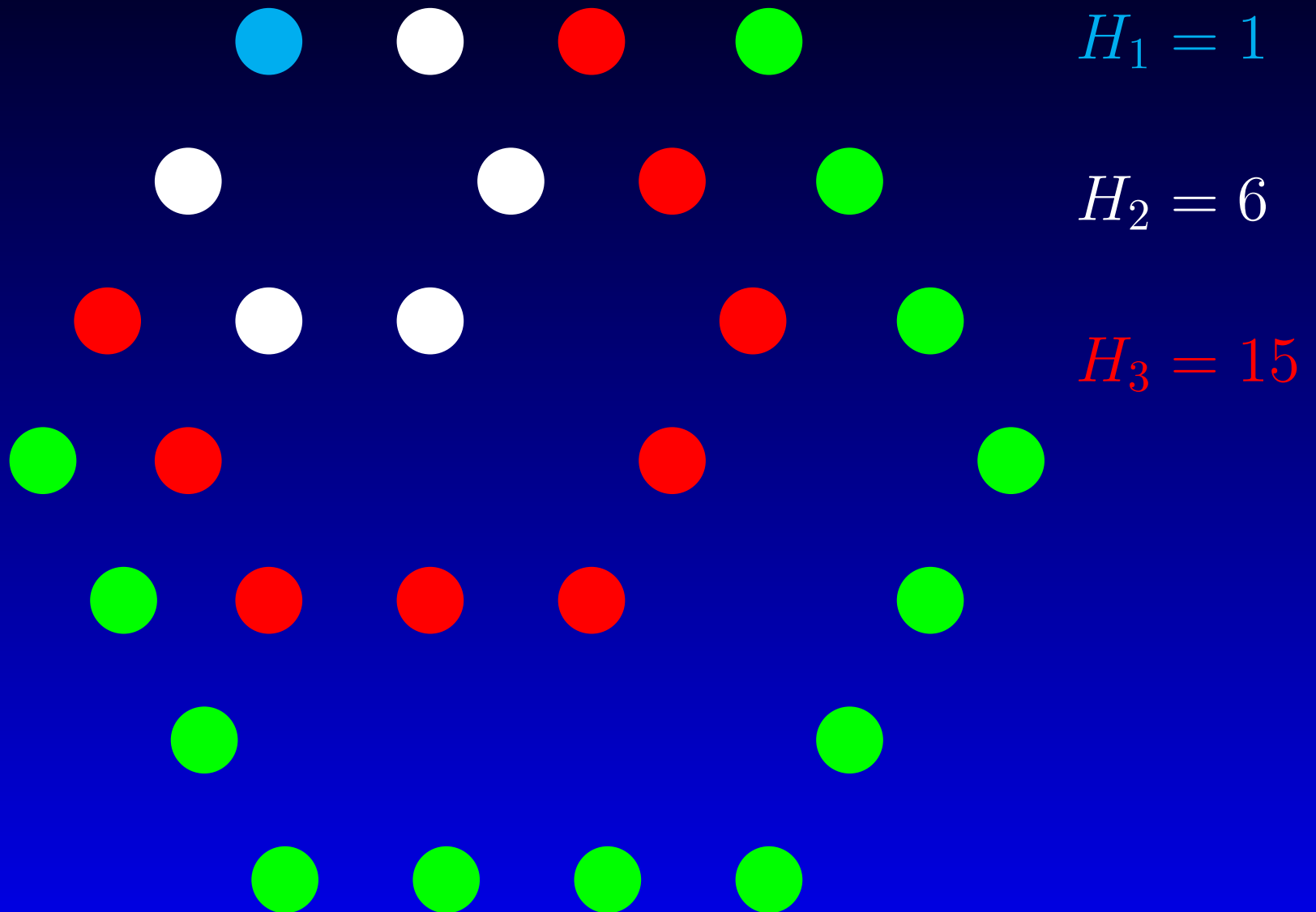
$$H_2 = 6$$



$$H_3 = 15$$



Hexagonal Numbers



Hexagonal Numbers

