

The Postage Stamp Problem

What postages can you pay using a combination of m -cent and n -cent stamps?

The Postage Stamp Problem

What postages can you pay using a combination of m -cent and n -cent stamps?

Is it true that you can obtain all postages after some number x ?

Example

Suppose $m = 7$ and $n = 5$.

Example

Suppose $m = 7$ and $n = 5$.
Obviously you cannot do

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	
11	
12	
13	
14	
15	
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	
12	
13	
14	
15	
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	Can't do
12	
13	
14	
15	
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	Can't do
12	$5 + 7$
13	
14	
15	
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	Can't do
12	$5 + 7$
13	Can't do
14	
15	
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	Can't do
12	$5 + 7$
13	Can't do
14	$2 \cdot 7$
15	
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	Can't do
12	$5 + 7$
13	Can't do
14	$2 \cdot 7$
15	$3 \cdot 5$
16	

The CAN-D0 Table for 5 and 7

Postage	How to get it
10	$2 \cdot 5$
11	Can't do
12	$5 + 7$
13	Can't do
14	$2 \cdot 7$
15	$3 \cdot 5$
16	Can't do

The CAN-D0 Table Continued

Postage	How to get it
17	
18	
19	
20	
21	
22	
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	
19	
20	
21	
22	
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	Can't do
19	
20	
21	
22	
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	Can't do
19	$2 \times 7 + 5$
20	
21	
22	
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	Can't do
19	$2 \times 7 + 5$
20	$4 \cdot 5$
21	
22	
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	Can't do
19	$2 \times 7 + 5$
20	$4 \cdot 5$
21	$3 \cdot 7$
22	
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	Can't do
19	$2 \times 7 + 5$
20	$4 \cdot 5$
21	$3 \cdot 7$
22	$3 \cdot 5 + 7$
23	

The CAN-D0 Table Continued

Postage	How to get it
17	$2 \cdot 5 + 7$
18	Can't do
19	$2 \times 7 + 5$
20	$4 \cdot 5$
21	$3 \cdot 7$
22	$3 \cdot 5 + 7$
23	Can't do

The CAN-D0 Table Finale

Postage	How to get it
24	
25	
26	
27	
28	
29	
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	
26	
27	
28	
29	
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	
27	
28	
29	
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	$3 \cdot 7 + 5$
27	
28	
29	
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	$3 \cdot 7 + 5$
27	$4 \cdot 5 + 7$
28	
29	
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	$3 \cdot 7 + 5$
27	$4 \cdot 5 + 7$
28	4×7
29	
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	$3 \cdot 7 + 5$
27	$4 \cdot 5 + 7$
28	4×7
29	$3 \cdot 5 + 2 \cdot 7$
30	

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	$3 \cdot 7 + 5$
27	$4 \cdot 5 + 7$
28	4×7
29	$3 \cdot 5 + 2 \cdot 7$
30	$6 \cdot 5$

The CAN-D0 Table Finale

Postage	How to get it
24	$2 \cdot 7 + 2 \cdot 5$
25	$5 \cdot 5$
26	$3 \cdot 7 + 5$
27	$4 \cdot 5 + 7$
28	4×7
29	$3 \cdot 5 + 2 \cdot 7$
30	$6 \cdot 5$

Can we keep going forever?

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

3 and 5

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

3 and 5

5 and 8

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

3 and 5

5 and 8

7 and 11

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

3 and 5

5 and 8

7 and 11

6 and 10

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

3 and 5

5 and 8

7 and 11

6 and 10

3 and 7

Analysis of 5 and 7 Problem

23 is the last postage we cannot make.

We can make all postages ≥ 24

What happens if the postages are:

3 and 5

5 and 8

7 and 11

6 and 10

3 and 7

This last case can be thought of as finding all football scores in which the scoring is done by field goals (3 points) or touchdowns with a successful point after (7 points). (No safties or 2-point conversions.)

Summarize Your Findings

m	n	Last Exception
5	7	
3	5	
5	8	
7	11	
6	10	
3	7	

Summarize Your Findings

m	n	Last Exception
5	7	23
3	5	
5	8	
7	11	
6	10	
3	7	

Summarize Your Findings

m	n	Last Exception
5	7	23
3	5	
5	8	
7	11	
6	10	
3	7	

Can you find a formula for the last exception in terms of m and n ?