

MATH 229 LECTURE NOTES: POWER RULES

Professor Richard Blecksmith
richard@math.niu.edu

1. DEFINITION OF DERIVATIVE

The **derivative** of the function $f(x)$ is defined to be

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2. DERIVATIVE OF A CONSTANT FUNCTION

Suppose $f(x) = c$, a constant.

To find $f'(x)$:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ &= \lim_{h \rightarrow 0} 0 \\ &= \boxed{0} \end{aligned}$$

3. TWO FAMILIAR FUNCTIONS

We have used the limit definition to evaluate the derivative of the following two functions.

Let $f(x) = x$.

Then $f'(x) = 1$.

The line $y = x$ has slope 1. It is not surprising that the tangent line to the line $y = x$ also has slope 1.

Let $f(x) = x^2$.

Then $f'(x) = 2x$.

4. EXPANDING CUBES

We will need to expand $(x + h)^3$

$$\begin{aligned}(x + h)^3 &= (x + h)(x + h)^2 \\ &= (x + h)(x^2 + 2xh + h^2) \\ &= \begin{array}{c} x^3 + 2x^2h + xh^2 \\ x^2h + 2xh^2 + h^3 \end{array} \\ &= x^3 + 3x^2h + 3xh^2 + h^3\end{aligned}$$

5. DERIVATIVE OF x^3

By the definition of derivative, for $f(x) = x^3$:

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x + h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3x^2 + 3xh + h^2)h}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\ &= 3x^2 + 3x \cdot 0 + 0^2 = \boxed{3x^2}\end{aligned}$$

6. TABLE OF DERIVATIVES

$f(x)$	$f'(x)$
x	1
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
x^5	$5x^4$
x^6	$6x^5$

POWER RULE: the derivative of x^n is nx^{n-1}

7. WARNING

- Knowing the power rule does not mean you **know** calculus
- Any more than knowing the quadratic formula means you know algebra
- But it is a start and . . .
- It is safe to say that you cannot know calculus if you do not know the power rule

8. TIME OUT FOR NOTATION UPDATE

- It is awkward to say “the derivative of x^n is nx^{n-1} ”
- Using the prime notation for derivatives, we could write
- $(x^n)' = nx^{n-1}$
- But most mathematicians prefer to use an alternative notation due to the Leibnitz, one of the co-inventors of calculus, along with
- Issac Newton, who favored the prime notation

9. LEIBNITZ NOTATION

- In the Leibnitz notation, if $y = f(x)$, then the derivative of $f(x)$ is written $\boxed{\frac{dy}{dx}}$
- or sometimes $\frac{d}{dx}y$

- or even $\frac{d}{dx}f(x)$
- Thus the power rule can be expressed as

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

10. WHICH NOTATION SHOULD YOU USE

- As a rule of thumb, the prime notation is best when working with generic functions: f , g , etc.
- The Leibnitz notation is best when working with letters which represent physical quantities.
- For example, in the formula $A = \pi r^2$
 - A stands for area (of a circle)
 - r stands for radius
 - π is the number 3.1415926 . . .
- The formula $\frac{dA}{dr} = 2\pi r$ tells us
- the area is increasing by a factor of 2π times the radius

11. NEGATIVE AND FRACTIONAL POWERS

Many important function in algebra can be written as powers if we allow negative exponents or fractional exponents.

For example

$$\frac{1}{x} = x^{-1}$$

$$\frac{1}{x^2} = x^{-2}$$

$$\sqrt{x} = x^{1/2}$$

12. NEW USES FOR THE POWER RULE

The good news is that the power rule still holds true when the exponent is negative or a fraction. Thus,

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}x^{-1} = (-1)x^{-2}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}x^{-2} = (-2)x^{-3}$$

$$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2}$$