

5.3 THE SHELL METHOD

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1. SHELL METHOD

An alternative method for finding volumes of solids of revolution uses a cylindrical shell formed by revolving a rectangle parallel to the axis of rotation.

To find the area A of a cylinder of height h and radius r :

unroll the cylinder to form a rectangle of length $2\pi r$ the circumference of the circle and width h

$$A = 2\pi r h$$

2. SHELL METHOD FORMULAS

If the axis of rotation is vertical (typically the y -axis), the volume of the solid of revolution is

$$V = \int_a^b 2\pi r h \, dx$$

Typically, r is given in terms of x and h is given in terms of y .

If the axis of rotation is horizontal (typically the x -axis),

$$V = \int_c^d 2\pi r h \, dy$$

In this case, r is usually given in terms of y and h is given in terms of x .

3. EXAMPLE 1

Use the shell method to find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and $0 \leq x \leq 1$ about the y -axis.

4. EXAMPLE 2 VOLUME OF SPHERE PART DEUX

Use the shell method to find the volume of a sphere of radius a .

We can obtain a sphere by rotating the semicircle about the y -axis.

For a circle of radius 17, $x^2 + y^2 = 17^2$

Upper semicircle has equation: $y = \sqrt{17^2 - x^2}$

5. EXAMPLE 2 CONTINUED

In the Shell Method, we rotate a thin rectangle about the y -axis to form a cylinder.

The radius of this cylinder is

$$r = x$$

The height of this cylinder is

$$h = 2\sqrt{17^2 - x^2}$$

6. EXAMPLE 2 FINALE

By the Shell Method, the volume of the sphere obtained by rotating the semicircle about the y -axis is

$$V = \int_a^b 2\pi r h dx$$

Plugging in $a = 0$, $b = 10$, $r = x$, $h = 2\sqrt{17^2 - x^2}$ gives

$$V = \int_0^{10} 2\pi x 2\sqrt{17^2 - x^2} dx$$

Can you finish this?