

1. MATH 353

Side-Side-Side Triangle Congruence Criterion

Professor Richard Blecksmith

richard@math.niu.edu

Dept. of Mathematical Sciences

Northern Illinois University

2. SIDE-SIDE-SIDE

Hypothesis:

In the correspondence $ABC \leftrightarrow XYZ$, assume

$$AB = XY$$

$$AC = XZ$$

$$BC = YZ$$

Then $\triangle ABC$ is congruent to $\triangle XYZ$

3. BEGINNING OF PROOF

If $\angle A = \angle X$, then the triangles are congruent by SAS and we are done.

So assume $\angle A \neq \angle X$.

WLOG we may assume $\angle A > \angle X$.

4. DETERMINE POINT D

By the Protractor (or Real Fan) Axiom there is a ray h with endpoint A such that

$$\overrightarrow{AB} - h - \overrightarrow{AC}$$

$$\text{and } m(\overrightarrow{AB}, h) = m(\angle X)$$

Ray h meets $\overline{BC}$ at a point D .

Why? **Crossbow Theorem.**

Idea is that B and C lay on opposite sides of the line $h \cup h'$.

See the previous section of book for details.

$$\angle BAD = \angle X$$

5. DETERMINE POINT P

By the Ruler (Real Ray) Axiom, there is a point P on $\overrightarrow{AD}$ such that $AP = XZ$.

Claim 1. $P \neq D$

Otherwise, $P = D$

$$\implies \triangle ABD \cong \triangle XYZ \text{ by SAS}$$

$$\implies BD = YZ = BC, \text{ contradiction.}$$

So either Case (i) $A - P - D$ or Case (ii) $A - D - P$.

We consider Case (i).

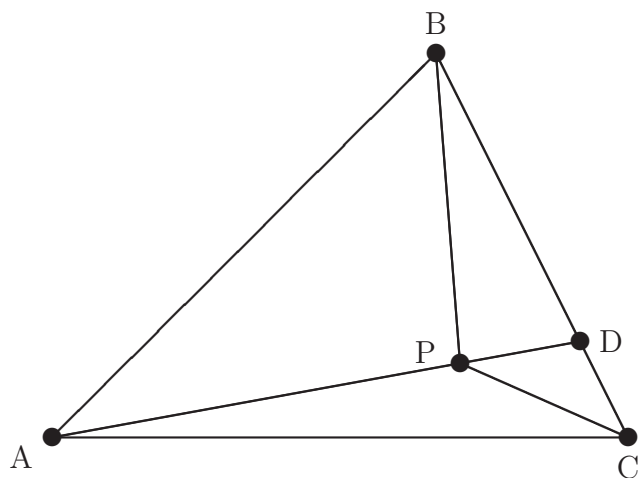
Case (ii) is Exercise 12.

6. DIAGRAM

Assume $A - P - D$

$$\triangle ABP \cong \triangle XYZ \text{ by SAS}$$

Draw segment $\overline{PC}$



7. TWO ISOCELES TRIANGLES

Claim 2: $\triangle PBC$ is isosceles

Why? $BP = YZ = BC$

So $\angle BPC = \angle BCP$

Claim 3: $\triangle PAC$ is isosceles

Why? $AP = XZ = AC$

So $\angle APC = \angle ACP$

8. COUP DE GRAS

$\angle ACB$

$= \angle ACP + \angle BCP$

$= \angle APC + \angle BPC$

$> \angle APC + \angle DPC$

$= 180$, contradiction.

This proves the theorem.