

MATH 303 FINAL EXAM STUDY GUIDE

1. Handouts

Module 3. Exploring Fractions

Mediants

Victoria's Theorem

Contiguous Fractions

Farey sequence of order n

Module 4. Mathematical Heresy

Fibonacci Numbers

$\sqrt{2}$ and Fermat's Method of Descent

Pythagorean Triples

To Infinity and Beyond

Counting Fractions

Irrationals—the Dark Matter of the Number Line

$\sqrt{-1}$ and Complex Numbers

Writing a prime as a sum of two squares

2. Definitions

mediant $\frac{a}{b} \oplus \frac{c}{d}$

contiguous fraction

Farey sequence $\mathcal{F}_n$

pseudo-prime base 2

Fibonacci number F_n

Type I prime

Type II prime

Pythagorean triple

3. Theorems

Victoria's Theorem: the mediant $\frac{a}{b} \oplus \frac{c}{d}$ lies between a/b and c/d

Mediant $\frac{a}{b} \oplus \frac{c}{d}$ is contiguous with a/b and c/d

Least Denominator Theorem [Thm 82]

The square root of 2 is irrational

descent argument

standard argument

Fermat's Little Theorem (base 2)

If p is a prime, then $2^{p-1} \bmod p = 1$.

Fibonacci Numbers

If a prime p ends in 1 or 9, then p divides F_{p-1} .

If p ends in 3 or 7, then p divides F_{p+1} .

4. Algorithms

Find the GCD of two numbers using the POW (or Euclidean) Algorithm

Find the Farey sequence recursively

Find the number adjacent to a fraction in the Farey sequence (using pails of water)

Find the Fibonacci sequence recursively

Distinguish between Type I and Type II primes

Write a fraction as a repeating decimal (or binary) number

Write a repeating decimal (or binary) number as a fraction

Count the rationals, using ordered pairs

Determine whether a prime is a sum of two squares