

5.2 THE WASHER METHOD

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1. VOLUME OF SOLID REGIONS

$$\text{Volume} = \int_a^b A(x) dx$$

where $A(x)$ is the cross-sectional area.

Think of slicing a loaf of bread.

2. SOLIDS OF REVOLUTION

When a region between two curves is rotated about an axis, the cross-sectional area is the area between two circles.

Say the bigger circle has radius R and the smaller circle has radius r .

This region is called a **washer**.

Then Cross-sectional area $= \pi R^2 - \pi r^2$

So the volume of the solid generated by rotating the region about a horizontal axis becomes

$$V = \int_a^b \pi R^2 - \pi r^2 dx$$

3. VERTICAL AXIS

If we rotate the region R about a vertical axis, the formula becomes the cross-sectional area is the area between two **horizontal** circles.

The volume of the solid generated by rotating the region about the vertical axis becomes

$$V = \int_c^d \pi R^2 - \pi r^2 dy$$

Here (a, b) and (c, d) are the intersection point of the two curves that form the region.

R and r must be given in terms of y .

4. SPECIAL CASE

When the smaller radius $r = 0$, the washer is just a disc of radius R . The cross-sectional area is just the area of this circle πR^2 .

The volume of the solid generated by rotating the region becomes

$$V = \int_a^b \pi R^2 dx$$

This formula is called the **disc method**.

5. EXAMPLE 1

Find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$ and $y = 1$, and $x = 0$ about the x -axis.

6. EXAMPLE 2

Find the volume of the solid of revolution formed by revolving the region bounded by $y = -x^2 + x$ and $y = 0$ about the x -axis.

7. EXAMPLE 3

Suppose the region in Example 2 is rotated about the line $y = 1$ (instead of the x -axis).

8. EXAMPLE 4 (2 INTEGRALS REQUIRED)

Find the volume of the solid of revolution formed by revolving the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y -axis.

9. EXAMPLE 5 (GEOMETRY FORMULA)

Find the volume of a sphere of radius 17.

We can obtain a sphere by rotating the semicircle about the x -axis.

For a circle of radius 17, $x^2 + y^2 = 17^2$

Upper semicircle has equation: $y = \sqrt{17^2 - x^2}$

which we rotate about x -axis.

10. TWO POINTS

In the Reference Section at the back of your text, page 1, we find

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$

Point 1. Why don't we just look up the answer?

What's A ?

What is the calculus connection between V and A ?

What is the calculus connection between A and C , the area and circumference of a circle?

Point 2. Is this a coincidence?